Homework 5

Newton's Law of Universal Gravitation

What is Gravity?

Gravity is the invisible force that **pulls everything together**. It keeps your feet on the ground, makes apples fall from trees, and keeps the planets moving around the Sun.

More than 300 years ago, **Sir Isaac Newton** discovered that gravity works not just on Earth but everywhere in the Universe.

Sir Isaac Newton (1643-1727).

Newton's Law of Gravitation

Newton's famous law can be written like this:

$$\left|F_{gravity}\right| = G\frac{m_1 m_2}{r^2} \quad (1)$$

Where:

- **F** – gravitational force (in newtons, N) between two objects with masses $\mathbf{m_1}$, and $\mathbf{m_2}$
- **r** – distance between the centers of the objects (in meters, m)
- **G** – gravitational constant: $G=6.67\times10^{-11}$ $N\cdot m^2/kg^2$

This number is very small, which is why you don't feel the pull of gravity from nearby people — only very massive objects like the Earth have a noticeable effect.

How hard does the Earth pull on a person?

Suppose a person has a mass of $\mathbf{m_1}$=60kg. The Earth's mass is $\mathbf{m_2}$=5.97×10^{24}kg

and its radius is **r**=6.37×10^{6}m.

This gives:

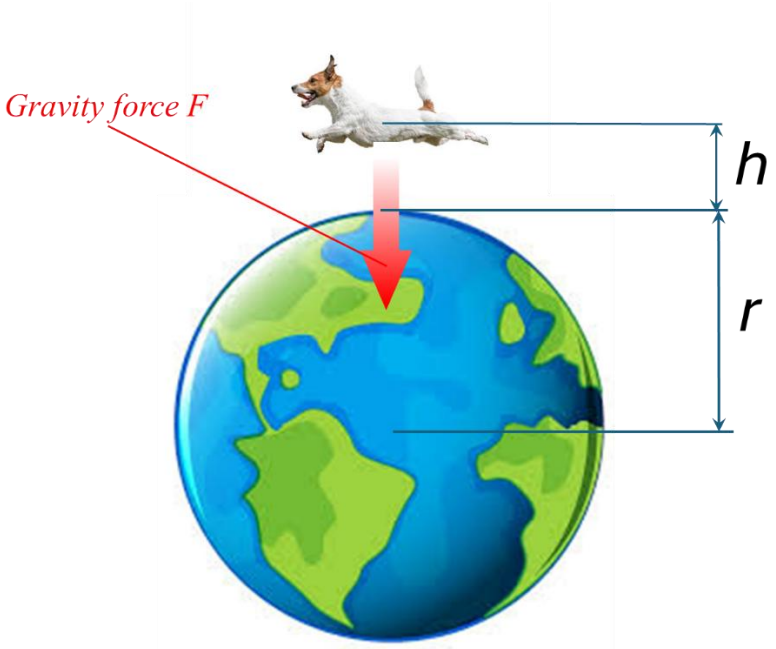
$$F = 6.67 \cdot 10^{-11}\mathrm{N} \cdot (\mathrm{m2/kg2}) \cdot \frac{60(\mathrm{kg}) \cdot 5.97 \cdot 10^{24}(kg)}{(6.37 \cdot 10^{6}(\mathrm{m}))^{2}} \approx 588N \qquad (2)$$

That is exactly the person's **weight** — the force with which the Earth pulls the person down!

Why do we use F=mg formula? It does not look like expression (1).

The expression $F = mg$ is approximal and can be used only if the object is close to the Earth surface. To be more specific, the distance between the object and the Earth surface has to be much less than the radius of Earth (**r**=6.37×10^{6}m).

Let us consider an object at an elevation ***h*** over the Earth surface (Figure 2)



The gravity force applied to the object can be calculated using the expression (1). But what we should use as a distance? We should use the distance between the centers of the object and Earth, which is ***r+h***. So, we have:

$$F_{gravity} = G\frac{m_1 m_2}{(r+h)^2} \qquad (3)$$

If ***h*** is much less than ***r,*** then we can just neglect it:

$$F_{gravity} \approx G\frac{m_1 m_2}{r^2} \equiv m_1 g \qquad (4)$$

Here, $g = G\frac{m_2}{r^2}$ (5).

The calculations give:

$$g = 6.67 \cdot 10^{-11}\text{N} \cdot \left(\frac{m^2}{kg^2}\right) \cdot \frac{5.97{\cdot}10^{24}(kg)}{\left(6.37{\cdot}10^{6}(\text{m})\right)^2} \approx 9.8\frac{m}{s^2}.$$

- **Units check:** G has units $(Nm^2)/kg^2$. After multiplying by m_2 (kg) and dividing by r^2, (m^2) the result is newtons per kilogram, which is the same as **m/s^2**.
- **Altitude matters:** Higher altitude → larger r+h → smaller g.
- **Location matters a bit:** Earth isn't a perfect sphere and it rotates, so g varies slightly (about 9.789 to 9.83 m/s^2).
- **Weight vs. mass:** Your **mass** (kg) doesn't change, but your **weight** W=mg changes if g changes (e.g., on the Moon).

Problems:

Problem 1 – Weight of an Apple
An apple has a mass of 0.1 kg. Using g=9.8 m/s^2, find its weight (the gravitational force Earth exerts on it).

Problem 2 – Tiny Gravity Between Two People
Two students, each of mass 50 kg, stand 1 m apart. Use $G=6.67\times10^{-11}$ $N{\cdot}m^2/kg^2$ to find the gravitational force between them.
(Hint: It will be *very* small!)

Problem 3 – Moon in Orbit
If gravity is always pulling the Moon toward Earth, why doesn't the Moon fall straight down?
(Hint: Think about the Moon's sideways motion).

Problem 4 g on the Moon.
Calculate acceleration due to gravity near the Moon surface. Moon mass is 7.35×10^{22} kg, Moon radius is 1.74×10^{6} m.