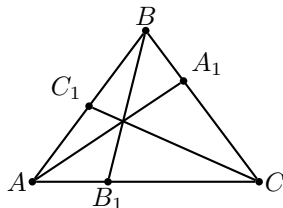


MATH 9
ASSIGNMENT 4: CEVA THEOREM AND MORE
OCT 18, 2026

CEVA THEOREM

Theorem. Let points A_1, B_1, C_1 be on the sides BC, AC, AB respectively of triangle ABC . Then lines AA_1, BB_1, CC_1 intersect at a single point if and only if

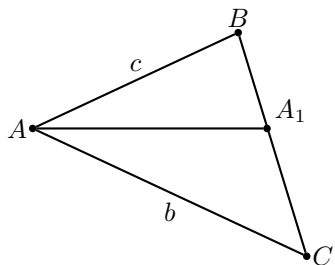
$$\frac{CA_1}{BA_1} \cdot \frac{BC_1}{AC_1} \cdot \frac{AB_1}{CB_1} = 1$$



PROBLEMS

1. Let $ABCD$ be a convex quadrilateral, and let points K, L, M, N be midpoints of sides AB, BC, CD , and DA respectively. Let O be the intersection point of segments KM and LN . Prove that O is the midpoint of each of them.
2. Let AA_1 be the bisector of angle A in triangle ABC . Show that then, A_1 divides side BC in the proportion equal to ratio of sides $c = AB, b = AC$:

$$|BA_1| : |CA_1| = c : b$$



[Hint: drop perpendiculars BB_1, CC_1 from vertices B, C to line AA_1 . Use similar triangles to show that $BB_1 : CC_1 = c : b$.]

3. Use previous problem and Ceva theorem to give another proof of the fact that the three angle bisectors of a triangle intersect at a single point.
4. Let A, B be distinct points. Show that then a point M lies on line AB if and only if one can write

$$\vec{OM} = t \vec{OA} + (1 - t) \vec{OB}$$

for some real t .

In particular, M lies on **segment** AB if and only if it can be written in the form above with $0 \leq t \leq 1$.

[Hint: write $\vec{OM}$ as combination of $\vec{OA}$ and $\vec{AB}$.]

5. Points A, B are moving along sides of an angle XOY so that

$$\frac{p}{OA} + \frac{q}{OB} = 1$$

stays constant. [Here p, q are some fixed positive real numbers.]

Prove that then there is a point M inside this angle such that for all such positions of A, B , the line AB goes through M .

6. Let R be the operation of rotation by angle 60° : for any vector $\vec{v}$, $R\vec{v}$ is a vector of same length as $\vec{v}$ but rotated 60 degrees counterclockwise.

(a) Show that $R(c\vec{v}) = cR(\vec{v})$, $R(\vec{v} + \vec{w}) = R(\vec{v}) + R(\vec{w})$.

(b) Given a triangle ABC , build on the outside of side AB an equilateral triangle. Denote by C_1 the center of this triangle.

Write vector $\vec{OC_1}$ as combination of $\vec{OA}$, $\vec{OB}$, $\vec{OC}$ and operation R .

- *(c) Repeat the same operation with two other sides of triangle ABC , building equilateral triangles on sides BC , AC . Show that the centers of the three constructed triangles themselves form an equilateral triangle.

[Hint: it suffices to show that $R(\vec{A_1B_1}) = \vec{B_1C_1}$.]