

MATH 8

PASCAL'S TRIANGLE AND COUNTING MODELS — FULL 2026–27

Pascal's Triangle

One of the most beautiful and useful structures in mathematics is **Pascal's triangle**, named after the French mathematician Blaise Pascal (1623–1662), though it was known to mathematicians in China, Persia, and India centuries earlier. At first glance it's just a triangular arrangement of numbers, each one the sum of the two above it. But hidden inside this single picture is essentially every important fact about the binomial coefficients $\binom{n}{k}$ — their algebraic formula, their recursive structure, the binomial theorem, and a long list of identities we will explore throughout the unit. The remainder of this handout is a slow tour of what's there.

Historical Note

The triangle of numbers we now call Pascal's was rediscovered many times before Pascal. It appears in a 10th-century commentary by the Indian scholar Halayudha on Pingala's earlier Sanskrit work on poetic meters; in the Persian mathematical tradition associated with al-Karaji and Omar Khayyam (c. 1100); and in the work of Jia Xian and Yang Hui in China (11th and 13th centuries). It is still often called *Yang Hui's triangle* in China and *Khayyam's triangle* in Iran. What made Pascal's *Traité du triangle arithmétique*, written around 1654 and published after his death in 1665, different was that it collected many separate properties of the triangle into a single coherent study. In that same year, Pascal's correspondence with Pierre de Fermat about gambling problems helped lay the foundations of probability theory. Pascal was a child prodigy who, by the age of nineteen, had invented one of the first mechanical calculators (the *Pascaline*); he was also a pioneering physicist and a deeply religious philosopher whose *Pensées* are still widely read today.



Blaise Pascal
(1623–1662)

Constructing the Triangle

Pascal's triangle is built row by row, starting with a single 1 at the top:

Row 0:								1														
Row 1:								1		1												
Row 2:								1		2		1										
Row 3:								1		3		3		1								
Row 4:								1		4		6		4		1						
Row 5:								1		5		10		10		5		1				
Row 6:								1		6		15		20		15		6		1		
Row 7:								1		7		21		35		35		21		7		1

The Construction Rule

The triangle has a strikingly simple recipe: every row begins and ends with 1, and each entry inside is the **sum of the two entries directly above it**.

$$\begin{array}{cc} a & b \\ \swarrow & \searrow \\ a+b \end{array}$$

For example, in Row 4, the entry 6 comes from $3 + 3$ (the two 3's in Row 3).

A Suspicious Coincidence

Now go back to Handout 1. There, $\binom{n}{k}$ was the number of ways to choose k people out of n . Let's compute all of them for $n = 4$:

$$\binom{4}{0} = 1, \quad \binom{4}{1} = 4, \quad \binom{4}{2} = 6, \quad \binom{4}{3} = 4, \quad \binom{4}{4} = 1.$$

That is exactly Row 4 of the triangle! Try $n = 5$: the numbers $\binom{5}{0}, \binom{5}{1}, \dots, \binom{5}{5}$ are 1, 5, 10, 10, 5, 1 — Row 5 again.

This should surprise you. The triangle was built by a mindless rule — *add two neighbors* — that knows nothing about people or committees. Binomial coefficients come from counting. Why on earth should the two agree? If the pattern holds forever, then the triangle is really a table of *all* the binomial coefficients, with $\binom{n}{k}$ sitting in Row n , Position k (both counted from 0):

$$\begin{array}{ccccccccc} & & & & \binom{0}{0} & & & & \\ & & & & \binom{1}{0} & & \binom{1}{1} & & \\ & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \\ \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \end{array}$$

What Would It Take?

Think about how the triangle is built. Once you know

- the **edges**: every row starts and ends with 1, and
- the **rule**: every inside entry is the sum of the two entries above it,

the whole triangle is completely determined — there are no choices left. Row 0 forces Row 1, Row 1 forces Row 2, and so on forever.

So to explain the coincidence, we only have to check that binomial coefficients obey the same edges and the same rule. The edges are easy: $\binom{n}{0} = 1$ (there is exactly one way to choose nobody) and $\binom{n}{n} = 1$ (one way to choose everybody). The real question is the rule. Written with binomial coefficients, “an entry in Row n is the sum of the two entries above it in Row $n - 1$ ” says

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

In words: *the number of ways to choose k people from n people can be computed from the ways of choosing from $n - 1$ people.* Why would that be true?

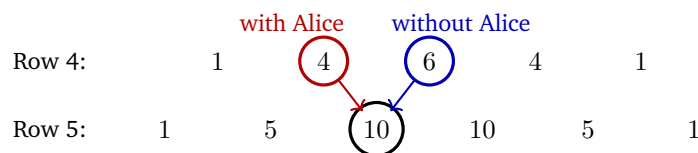
Pascal's Identity

The Idea: Keep an Eye on One Person

Start small. How many 2-person committees can be chosen from five people — Alice, Bob, Carol, Dan, and Eve? We already know the answer is $\binom{5}{2} = 10$. But let's count them in a new way, sorting the committees by a single question: *is Alice on it?*

Alice is on the committee	Alice is not on the committee
AB, AC, AD, AE	BC, BD, BE, CD, CE, DE
choose her 1 partner from the other 4	choose both members from the other 4
$\binom{4}{1} = 4$	$\binom{4}{2} = 6$

Every committee lands in exactly one column, so $10 = 4 + 6$, that is, $\binom{5}{2} = \binom{4}{1} + \binom{4}{2}$. Now look at the triangle: the 10 in Row 5 sits directly below the 4 and the 6 in Row 4!



The Combinatorial Proof

Nothing in that argument depended on the numbers 5 and 2.

Pascal's Identity

For $1 \leq k \leq n - 1$,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

Proof. Count the k -person committees chosen from a group of n people; there are $\binom{n}{k}$ of them. Pick one person in the group — call her Alice — and sort the committees into two piles.

- **Alice is on the committee.** Her seat is taken, so we fill the other $k - 1$ seats from the remaining $n - 1$ people: $\binom{n-1}{k-1}$ ways.
- **Alice is not on the committee.** Then all k seats go to the remaining $n - 1$ people: $\binom{n-1}{k}$ ways.

Every committee is in exactly one pile, so $\binom{n}{k}$ is the sum of the two. $\square$

Here is what the proof is really saying. Row $n - 1$ of the triangle describes the world *before* Alice arrives: its entries count committees chosen from $n - 1$ people. Row n describes the world *after* Alice joins. Every new committee either leaves Alice out — then it is simply an old committee of the same size (the entry above on the right) — or includes Alice — then it is an old committee with one fewer member, plus Alice (the entry above on the left). So

adding the two numbers above = counting committees without Alice + committees with Alice,

and **moving one row down the triangle means one more person has joined the group**. The construction rule was secretly a counting argument all along.

The Mystery Solved

Why Pascal's triangle is a table of binomial coefficients

The triangle and the binomial coefficients start the same way (Row 0 is a single 1), have the same edges (all 1s), and grow by the same rule (Pascal's identity). So they must agree by row: if Row $n - 1$ of the triangle is $\binom{n-1}{0}, \binom{n-1}{1}, \dots, \binom{n-1}{n-1}$, then applying the same rule to both produces the same Row n . Row 0 agrees, so Row 1 agrees, so Row 2 agrees, ... forever.

This “one row forces the next, forever” style of argument is a first taste of **mathematical induction**, which we will meet properly later in the year.

Invisible zeros. There is a cute way to make the edges obey the rule too. Imagine every row surrounded by invisible 0s:

$$\dots 0 \quad 1 \quad 3 \quad 3 \quad 1 \quad 0 \dots$$

Then each 1 on the edge of the next row is just $0 + 1$. This matches counting perfectly: there are 0 ways to choose -1 people, or to choose more people than you have. With the convention $\binom{n}{k} = 0$ when $k < 0$ or $k > n$, Pascal’s identity holds for every k .

A Second Proof: Algebra

The factorial formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ lets us check the identity by pure computation. Put both fractions over the common denominator $k!(n-k)!$:

$$\begin{aligned} \binom{n-1}{k-1} + \binom{n-1}{k} &= \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-1-k)!} \\ &= \frac{(n-1)! \cdot k}{k!(n-k)!} + \frac{(n-1)! \cdot (n-k)}{k!(n-k)!} \\ &= \frac{(n-1)!(k + (n-k))}{k!(n-k)!} = \frac{n!}{k!(n-k)!} = \binom{n}{k}. \end{aligned}$$

This proof is short and completely convincing — but it doesn’t tell you *why*: the factors simply happen to cancel. The counting proof explains where the identity comes from, and it works even if you have forgotten the formula. It is worth knowing both. Throughout this unit you will see identities proved in both styles, and the counting proof is usually the one that makes you say “of course!”

Quick Check

1. Verify Pascal’s identity for $n = 6$, $k = 3$.
2. Use Pascal’s identity to find $\binom{8}{4}$ if you know $\binom{7}{3} = 35$ and $\binom{7}{4} = 35$.
3. Six friends, one of them Alice, choose a 2-person team. How many teams include Alice? How many don’t? Which entries of the triangle are these, and where do they sit relative to $\binom{6}{2}$?
4. Write the next row (Row 8) of Pascal’s triangle.

Three Counting Models and One Recursive Representation

We now reach what may be the central conceptual moment of the whole unit. The same number $\binom{n}{k}$ counts three very different-looking collections: subsets, grid paths, and binary strings. It also appears as an entry generated recursively in Pascal’s triangle. Recognizing that “different descriptions, one number” is the kind of unification that makes combinatorics worth studying: mathematics is telling us that there is hidden structure connecting them.

Four useful descriptions of $\binom{n}{k}$

1. The number of ways to choose k items from n items (subsets).
2. The number of paths on a grid going k steps up and $n - k$ steps right.
3. The number of binary strings of length n with exactly k ones.

4. The entry in row n , position k of Pascal's triangle.

Let's explore each description in turn. Then we will build bijections among the three collections and explain why Pascal's recursive recipe produces the same numbers.

Interpretation 1: Counting Subsets

This is our original definition from Handout 1:

$$\binom{n}{k} = \text{number of } k\text{-element subsets of an } n\text{-element set.}$$

Example 1. How many 3-person committees can be formed from 5 people $\{A, B, C, D, E\}$?

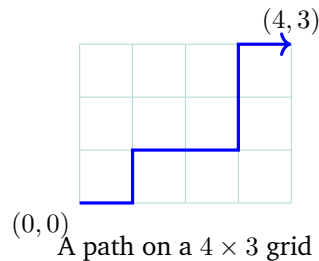
Solution. We list them: $\{A,B,C\}$, $\{A,B,D\}$, $\{A,B,E\}$, $\{A,C,D\}$, $\{A,C,E\}$, $\{A,D,E\}$, $\{B,C,D\}$, $\{B,C,E\}$, $\{B,D,E\}$, $\{C,D,E\}$.

That's $\binom{5}{3} = 10$ committees.

Interpretation 2: Counting Paths on a Grid

The second interpretation is geometric, and at first feels unrelated to the first. Imagine you are walking through the streets of a city laid out as a perfect square grid — and you can only walk east or north. How many distinct routes are there between two corners?

Consider a grid that extends m units to the right and n units upward. We want to count paths from the bottom-left corner $(0,0)$ to the top-right corner (m,n) , where we can only move **right (R)** or **up (U)**.



Key observation: Any path from $(0,0)$ to (m,n) consists of exactly m moves right and n moves up, in some order. The total number of moves is $m+n$.

Counting the paths: A path is determined by choosing which n of the $m+n$ moves are “up” (the rest are “right”). So:

$$\text{Number of paths} = \binom{m+n}{n} = \binom{m+n}{m}.$$

Example 2. How many paths go from $(0,0)$ to $(4,3)$?

Solution. We need 4 right moves and 3 up moves, totaling 7 moves. Choose which 3 are “up”:

$$\binom{7}{3} = \frac{7 \times 6 \times 5}{3!} = 35 \text{ paths.}$$

Example 3. How many shortest paths go from $(0,0)$ to $(5,5)$ that pass through $(2,3)$?

Solution. Break into two parts:

- $(0,0)$ to $(2,3)$: need 2 rights and 3 ups, so $\binom{5}{3} = 10$ paths.

- $(2, 3)$ to $(5, 5)$: need 3 rights and 2 ups, so $\binom{5}{2} = 10$ paths.

Total: $10 \times 10 = 100$ paths.

Quick Check

5. How many paths go from $(0, 0)$ to $(3, 4)$?
6. How many paths go from $(0, 0)$ to $(6, 6)$?
7. A path goes from $(0, 0)$ to $(5, 3)$. How many such paths pass through $(2, 2)$?

Interpretation 3: Counting Binary Strings

The third interpretation is the most basic of all. Strip a counting problem down to its bones and you often end up with a sequence of binary choices — yes/no, on/off, 0/1. Binary strings are how computers store information, so this interpretation is also the bridge between combinatorics and computer science.

A **binary string** is a sequence of 0s and 1s, like 01101 or 11000.

Question: How many binary strings of length n have exactly k ones?

Answer: Each such string is determined by choosing which k positions (out of n) contain a 1. So there are $\binom{n}{k}$ such strings.

Example 4. List all binary strings of length 4 with exactly 2 ones.

Solution. The positions of the 1s can be: $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2, 3\}$, $\{2, 4\}$, $\{3, 4\}$.

The strings are: 1100, 1010, 1001, 0110, 0101, 0011.

That's $\binom{4}{2} = 6$ strings.

Connection to Grid Paths

Notice that binary strings and grid paths are essentially the same thing!

A path on a grid can be encoded as a string: write R for “right” and U for “up.” For example, the path RRUURRR corresponds to starting at $(0, 0)$, going right twice, up twice, then right three times, ending at $(5, 2)$.

If we replace R with 0 and U with 1, we get a binary string: 0011000.

This string has length $5 + 2 = 7$ (total moves) and exactly 2 ones (up moves).

So both objects are counted by the same number:

$$\#(\text{Paths from } (0, 0) \text{ to } (m, n)) = \binom{m+n}{n} = \#(\text{Binary strings of length } m+n \text{ with } n \text{ ones}).$$

Quick Check

8. How many binary strings of length 8 have exactly 3 ones?
9. Write the binary string corresponding to the grid path RURRRU.
10. Write the grid path corresponding to the binary string 1100101.

Interpretation 4: Pascal’s Triangle (Recursive Definition)

The last description is the one we started with: $\binom{n}{k}$ is the entry in Row n , Position k of Pascal’s triangle. This one is different in kind — it isn’t a counting question at all, but a *recipe* for generating the numbers.

We can define $\binom{n}{k}$ recursively using Pascal’s identity:

- **Base cases:** $\binom{n}{0} = \binom{n}{n} = 1$ for all $n \geq 0$.
- **Recursive case:** $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ for $0 < k < n$.

This definition builds Pascal's triangle row by row, and automatically gives us the values of all binomial coefficients. The boundary values determine the edges, and then the recursive rule determines every remaining entry, one row at a time.

Why the Recipe Gives the Right Numbers

This is exactly the mystery we solved at the start of the handout: the recipe and the counting definition have the same edges and obey the same rule, so they produce the same numbers, row by row.

Notice that each counting model comes with its own version of the “keep an eye on Alice” argument. For subsets, we asked whether one special element is in the subset. For binary strings, you can ask what the *last bit* is; for grid paths, what the *last step* is. Each question splits the objects into two piles that are counted by the two entries in the row above. (You will carry these out in the homework.)

Why the Three Counting Models Agree: Bijections

We have three very different-looking collections that all contain $\binom{n}{k}$ objects. *Why?* The cleanest answer is not to compute three times and notice that the numbers match. It is to show that the collections are *the same collection in different costumes*: we can match them element by element, with nothing left over on either side.

Bijection proof

Two finite sets A and B are in **bijection** (a *one-to-one correspondence*) if you can pair up the elements of A with the elements of B so that:

- every element of A has exactly one partner in B ;
- every element of B has exactly one partner in A .

If such a pairing exists, then A and B have the same size. Proving that two sets are in bijection is one of the most powerful ways to show they have the same count — it works *without* ever computing either count.

Here are the bijections among our three counting models.

1. **Subsets** $\leftrightarrow$ **Binary Strings**. Given a k -element subset $S \subseteq \{1, 2, \dots, n\}$, write the length- n binary string with a 1 in position i if and only if $i \in S$. For example, with $n = 5$ and $S = \{1, 3, 4\}$, the matching string is 10110. The map “subset $\rightarrow$ string” is a bijection: every subset gives a unique string, and every string of length n with exactly k ones comes from a unique subset (read off which positions hold a 1).
2. **Binary Strings** $\leftrightarrow$ **Grid Paths**. Given a length- n binary string with k ones, read its characters left to right and translate $0 \mapsto$ “step right” and $1 \mapsto$ “step up.” This produces a lattice path from $(0, 0)$ to $(n - k, k)$. The map is a bijection: each string gives a unique path, and each path gives back a unique string (record an R or U at each step).

Chaining the two bijections gives a direct bijection “subset $\rightarrow$ path”: from $S \subseteq \{1, \dots, n\}$, build the path whose i -th step is up if $i \in S$ and right otherwise. Thus subsets, strings, and paths are three descriptions of the same underlying choices.

Pascal's triangle plays a different role. It is not a fourth set of objects, so there is no fourth bijection to construct. Instead, Pascal's identity and the boundary values $\binom{n}{0} = \binom{n}{n} = 1$ give a recursive way to compute the common number. The factorial formula gives an algebraic way to compute it. Our earlier proofs of Pascal's identity show that all these descriptions agree.

Why bijective proofs are powerful. Bijection is one of the central techniques in combinatorics, and you'll see it again throughout this curriculum:

- *Stars and bars* (Handout 5) is a bijection between distributions of identical objects and arrangements of dots and bars.
- The *reflection trick* for Catalan numbers (Bonus Handout) is a bijection between certain “bad” lattice paths and shifted paths.
- Pascal’s identity itself, which we proved combinatorially earlier in this handout, is fundamentally a bijective argument: it splits the k -subsets of $\{1, \dots, n\}$ into two groups — those that contain n (our “Alice”) and those that don’t — and matches each group with a family of subsets of $\{1, \dots, n-1\}$.

Once you start looking for bijections, you’ll find them everywhere.

Key Takeaways

- **Pascal’s Triangle:** Each entry is the sum of the two above it.
- **Pascal’s Identity:** $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$. *Why:* split the committees by whether Alice is on them.
- **The triangle is a table of binomial coefficients** because both have the same edges and follow the same rule, so they agree row by row.
- **Three equivalent counting models for $\binom{n}{k}$:**
 - Choosing k items from n (subsets)
 - Grid paths with k up-moves and $n-k$ right-moves
 - Binary strings of length n with k ones
- **Recursive representation:** $\binom{n}{k}$ is also the entry in row n , position k of Pascal’s triangle.
- These interpretations give multiple ways to think about the same problem.

Common Mistakes

- **Forgetting row/position numbering starts at 0:** Row 0 has one entry (1). The entry $\binom{4}{2} = 6$ is in Row 4, Position 2.
- **Grid path confusion:** For a path from $(0, 0)$ to (m, n) , you need m rights and n ups, totaling $m+n$ steps. The count is $\binom{m+n}{n}$ (or equivalently $\binom{m+n}{m}$). Don’t accidentally write $\binom{m}{n}$ — that confuses the total step count with one coordinate.
- **Mixing up the binary string correspondence:** Decide consistently what 0 and 1 represent (usually 0=right, 1=up).

Practice Problems

1. Write out Row 9 of Pascal’s triangle.
2. Use Pascal’s identity to compute $\binom{10}{4}$, given that $\binom{9}{3} = 84$ and $\binom{9}{4} = 126$.
3. How many shortest paths go from $(0, 0)$ to $(5, 4)$ on a grid? Verify by computing $\binom{9}{4}$.
4. List all binary strings of length 5 with exactly 2 ones. How many are there?
5. A committee of 4 is chosen from 9 people.
 - (a) How many committees are possible?
 - (b) Express this as an entry in Pascal’s triangle.
 - (c) Describe the binary strings that correspond to these committees and explain the correspondence.
6. How many paths from $(0, 0)$ to $(4, 4)$ pass through the point $(2, 2)$?

7. Prove combinatorially that $\binom{n}{k} = \binom{n}{n-k}$ (the symmetry identity) using the subset interpretation.

Practice Problems: Solutions

1. **Row 9 of Pascal's triangle:**

1, 9, 36, 84, 126, 126, 84, 36, 9, 1

(Computed using the addition rule from Row 8: 1, 8, 28, 56, 70, 56, 28, 8, 1)

2. **Using Pascal's identity:**

$$\binom{10}{4} = \binom{9}{3} + \binom{9}{4} = 84 + 126 = 210.$$

3. **Paths from (0, 0) to (5, 4):**

Need 5 rights and 4 ups, total 9 moves. Choose 4 for up: $\binom{9}{4} = \frac{9 \times 8 \times 7 \times 6}{4!} = \frac{3024}{24} = 126$.

4. **Binary strings of length 5 with 2 ones:**

10100, 10010, 10001, 01100, 01010, 01001, 00110, 00101, 00011, 11000.

Count: $\binom{5}{2} = 10$.

5. **Committee of 4 from 9 people:**

(a) $\binom{9}{4} = 126$ committees.

(b) This is the entry in Row 9, Position 4 of Pascal's triangle.

(c) Each committee corresponds to a binary string of length 9 with 4 ones (where 1 means "chosen").

6. **Paths from (0, 0) to (4, 4) through (2, 2):**

• (0, 0) to (2, 2): $\binom{4}{2} = 6$ paths.

• (2, 2) to (4, 4): $\binom{4}{2} = 6$ paths.

Total: $6 \times 6 = 36$ paths.

7. **Combinatorial proof of symmetry:**

$\binom{n}{k}$ counts k -element subsets of an n -element set.

$\binom{n}{n-k}$ counts $(n-k)$ -element subsets of an n -element set.

These are the same because choosing which k elements to **include** is equivalent to choosing which $n-k$ elements to **exclude**. Each k -subset corresponds to a unique $(n-k)$ -subset (its complement).

Homework

- How many paths are there from $(0, 0)$ to:
 - $(3, 5)$?
 - $(6, 6)$?
 - $(10, 2)$?
- How many binary strings of length 10 have:
 - exactly 4 ones?
 - at least 8 ones?
 - more ones than zeros?
- Prove that the sum of the entries in Row n of Pascal's triangle is 2^n . *Hint: think about binary strings.*
- Repair the triangle.** Row 9 of Pascal's triangle is 1, 9, 36, 84, 126, 126, 84, 36, 9, 1. Use only Pascal's rule to produce Row 10. Then explain why Row 10 reads the same forwards and backwards.
- Three costumes for one object.** Use $0 = R$ and $1 = U$.
 - For the subset $S = \{2, 5, 6\}$ of $\{1, 2, \dots, 8\}$, write the matching binary string and grid path.
 - For the binary string 10100110, write the matching subset and grid path.
- M Pascal's identity, another costume.** Prove that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ by counting binary strings of length n with exactly k ones. Sort them into two piles by their *last bit*. Which bit plays the role of Alice?
- Closed intersection.** A robot travels from $(0, 0)$ to $(7, 5)$ using only R- and U-moves, but the intersection $(3, 2)$ is closed for repairs. How many routes avoid it?
- The museum light board.** A display has 10 lamps, two of them red. Exactly 4 lamps must be on, but the two red lamps may not both be on. How many displays are possible?
- M** How many paths from $(0, 0)$ to $(6, 4)$ have different first and last steps? *Hint: the first and last steps are either $R \dots U$ or $U \dots R$.*
- H** A path from $(0, 0)$ to (n, n) is called **good** if it never goes above the diagonal $y = x$.
 - List all the good paths from $(0, 0)$ to $(3, 3)$. How many are there?
 - Can you find a formula for the number of good paths to (n, n) ? (These are the famous **Catalan numbers**.)
Hint: count the bad paths instead. A bad path first touches the line $y = x + 1$ somewhere; reflect the part of the path before that point across this line. Where does the reflected path start?
- M No two heads in a row.** Let $f(n)$ be the number of sequences of n coin tosses in which HH never appears. For example, $f(3) = 5$: TTT, TTH, THT, HTT, HTH. (We also set $f(0) = 1$ and $f(1) = 2$.)
 - Find $f(4)$ by listing.
 - Find a rule that gives $f(n)$ from $f(n-1)$ and $f(n-2)$. *Hint: look at the last toss — just like we looked at Alice.*
 - Find $f(12)$.

Quick Check Answers

- $\binom{5}{2} + \binom{5}{3} = 10 + 10 = 20 = \binom{6}{3}$. sit just above $\binom{6}{2} = 15$. 7. $\binom{4}{2}\binom{4}{1} = 24$.
- $\binom{8}{4} = 70$. 4. Row 8: 1, 8, 28, 56, 70, 56, 28, 8, 1. 8. $\binom{8}{3} = 56$.
- With Alice: $\binom{5}{1} = 5$; without: $\binom{5}{2} = 10$; the 5 and 10 in Row 5 5. $\binom{7}{4} = 35$. 9. 010001.
6. $\binom{12}{6} = 924$. 10. UURURU.