

(1) Compute:

$$\frac{2026}{202620262026^2 - 202620262025 \cdot 202620262027}.$$

(2) Find all pairs (x, y) such that

$$x^2 - 2000x = y^2 - 2000y, \quad x \neq y.$$

(3) Simplify using algebraic identities (do not expand all the multiplications):

(a) $(x + 7)^2 - (x - 7)^2$

(b) $(2a + 3)^2 - (2a - 3)^2$

(c) $(5y + 1)(5y - 1)$

(4) Rationalize the denominator:

(a) $\frac{1}{\sqrt{3} - 1}$

(b) $\frac{4}{2 + \sqrt{5}}$

(5) Expand and simplify:

(a) $(x + 2)^3$

(b) $(y - 5)^3$

(6) Simplify $\frac{1}{1 + \sqrt{2}} + \frac{1}{1 - \sqrt{2}}$.

(7) Simplify each expression.

(a) $\frac{42^2}{6^2}$

(b) $\frac{6^3 \times 6^4}{2^3 \times 3^4}$

(c) $(2^{-3} \times 2^7)^2$

(d) $\frac{3^2 \times 6^{-3}}{10^{-3} \times 5^2}$

(8) Expand the following expressions.

(a) $(4a - b)^2$

(b) $(a + 9)(a - 9)$

(c) $(3a - 2b)^2$

(9) Factor each expression.

(a) $ab + ac$

(b) $3a(a + 1) + 2(a + 1)$

(c) $36a^2 - 49$

(d) $a^9 - 27$

(10) Use algebraic identities to evaluate:

(a) $299^2 + 598 + 1$

(b) 199^2

(c) $51^2 - 102 + 1$

(11) Compute:

$$\frac{(2015 \cdot 2035 + 100)(2005 \cdot 2045 + 400)}{2025^4}$$

(12) Compute:

$$\left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \left(1 - \frac{1}{16}\right) \cdots \left(1 - \frac{1}{225}\right)$$

(13) Prove the following identity:

$$(a^2 + b^2)(u^2 + v^2) = (au + bv)^2 + (av - bu)^2$$

(14) Find the expansions of $(a + b)^4$ and $(a - b)^4$ using previous results.

(15) Rewrite each expression in the form $a + b\sqrt{3}$, where a and b are rational numbers.

(a) $(1 + \sqrt{3})^2$

(b) $(1 + \sqrt{3})^3$

(c) $\frac{1}{1 - 2\sqrt{3}}$

(d) $\frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

(e) $\frac{1 + 2\sqrt{3}}{\sqrt{3}}$