

MATH 7 HOMEWORK 4: Algebraic identities. Pythagorean theorem

October 18, 2026

1. Exponents Laws

If a and b are real non-negative numbers and n is a positive integer

a. $(ab)^n = a^n b^n$

b. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

c. $(a + b)^2 = a^2 + 2ab + b^2$

d. $(a - b)^2 = a^2 - 2ab + b^2$

e. $a^2 - b^2 = (a - b)(a + b)$

Replacing in the last equality a by $\sqrt{a}$, b by $\sqrt{b}$, we get:

f. $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

2. Simplifying expressions with roots (rational expressions)

The last identity above can be used to simplify expressions with roots by expanding the fractions with a term which "removes" the roots from the denominator:

$$\frac{1}{\sqrt{2} + 1} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

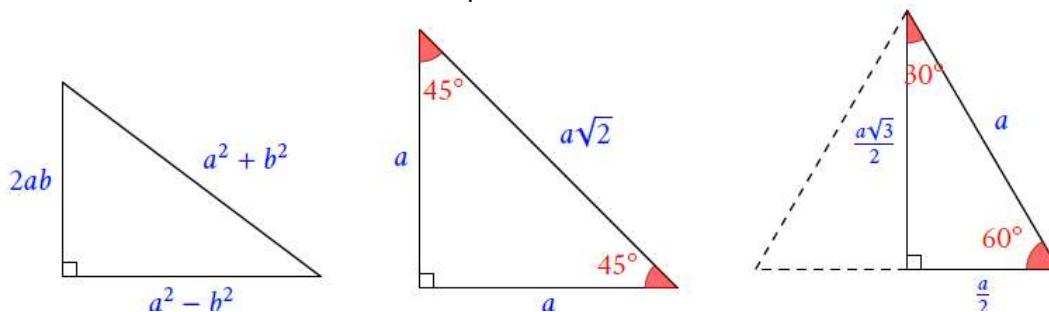
3. Quadratic equations of a specific form

- linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown and equation)
- two types of quadratic equations (i.e, equations where the unknown is squared, x^2) when the left-hand side could be factored as product of linear factors, i.e, $(x - 2)(x + 3) = 0$.

4. Pythagoras' theorem

In a right triangle with legs a and b , and hypotenuse c : $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $c^2 = a^2 + b^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61), (20,21,29).

To generate such Pythagorean triples, choose two positive integers a and b . Then plug the values into the sides as shown on the first picture:



Try to figure out again why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the angles in a right triangle is 45° , the other angle is also 45° , and two of its legs are equal.

If the length of a leg is a , the hypotenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is 30° , the other angle is 60° . Such triangle is a half of the

equilateral triangle. That means that if the hypotenuse is equal to a , its smaller leg is equal to the half of the hypotenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

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Instructions: Please always write solutions on a *separate sheet of paper*. Solutions should include explanations **how you arrived at this answer**.

1. Simplify

a. $\frac{6^3 \cdot (6^4)^2}{2^3 \cdot 3^4} =$

b. $(2^{-3} \cdot 2^7)^2 =$

c. $\frac{3^2 \cdot 6^{-3}}{10^{-3} \cdot 5^2}$

2. Simplify

a) $\frac{a}{2} + \frac{b}{4} + \frac{c}{8} =$

b) $\frac{1}{a} + \frac{1}{b} =$

c) $\frac{3}{x} + \frac{5}{xy} =$

3. Solve system of equations:

a. $\begin{cases} 6x - 5y = -3 \\ x + y = 5 \end{cases}$

b. $\begin{cases} 5x + 2y = 16 \\ 2x + 3y = 13 \end{cases}$

4. Using algebraic identities calculate

a. $299^2 + 598 + 1 =$

b. $199^2 =$

c. $51^2 - 102 + 1 =$

5. Expand preferably using your knowledge about algebraic identities

a. $(4a - b)^2 =$

b. $(a + 9)(a - 9) =$

c. $(3a - 2b)^2 =$

6. Solve the following quadratic equations. *Hint: Factor first (i.e., write as a product):*

a. $x^2 - 18x + 81 = 0$

b. $3x(x + 1) + 2(x + 1) = 0$

c. $36a^2 - 49 = 0$

7. Write each of the following expressions in the form $a + b\sqrt{3}$ with rational a, b. (No root in the denominator):

a. $(1 + \sqrt{3})^2$

b. $(1 + \sqrt{3})^3$

c. $\frac{1}{1-2\sqrt{3}}$

d. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

e. $\frac{1+2\sqrt{3}}{\sqrt{3}}$

8. In a trapezoid ABCD with bases AD and BC, $\angle A = 90^\circ$, and $\angle D = 45^\circ$. It is also known that $AB = 10$ cm, and $AD = 3BC$. Find the area of the trapezoid.

9. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of $BC=13$, and $AB=5$. What is the length of AD?