Math 6e: Homework 5

HW#5 is due October 28; submit to Google Classroom 15 minutes before class.

Please write clearly which problem you are solving and show all steps of your solution.

Logical variables are basic statements labeled with A, B, C, …, that can be evaluated as T or F.

For example, ***A = "Bill is a knight"*** and ***C = "Carl is a knight"*** can be evaluated (answered) as True or False.

Logical operations AND, OR, NOT combine variables (statements) in more complicated statements. A **truth table** for a logical operation evaluates all possible combinations of two or more variables that are combined by the logical operation.

Examples: A **AND** B = "Bill is a knight **AND** Carl is a knight". True only if A and B are both true

A **OR** B = "Bill is a knight **OR** Carl is a knight". True if at least one or both of the variables are True

NOT A = " Bill is not a knight". Inverts the answer.

Exclusive OR: (A **XOR** B). True if only one is True, false if the same.

IF → THEN: **A → B**. Reads "If A, then B". If A is false, it is automatically true; if A is true, it is true only when B is true

Truth tables:

A	B	A **AND** B
T	T	T
T	F	F
F	T	F
F	F	F

A	B	A **XOR** B
T	T	F
T	F	T
F	T	T
F	F	F

A	**NOT** A
T	F
F	T

A	B	A **OR** B
T	T	T
T	F	T
F	T	T
F	F	F

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

Logic Laws: We can combine logic operations to create more complicated expressions

Here are two other laws: $NOT(A\ AND\ B)\ is\ the\ same\ (NOTA)\ OR(NOTB)$

$A \rightarrow B\ is\ the\ same\ as\ (NOTB) \rightarrow (NOTA)$

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.

Instructions: In problems 1 and 2, you need to (a) write the obvious conclusion from the given statements; and (b) justify the conclusion by writing a chain of arguments that leads to it. It may help to write the given statements and conclusion by logical formulas (denoting which are used by letters A, B, … connected by logical operations OR, AND, …).

Homework questions

1. Prove that: A→ B = NOT (A AND (NOT B))

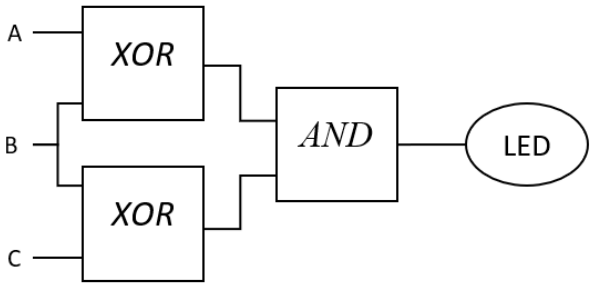
2. If today is Thursday, then Jane's class has library day. If Jane's class has a library day, then Jane will bring home new library books. Jane brought no new library books. Therefore …

3. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore …

4. You probably know Lewis Carroll as the author of Alice in Wonderland and other books. What you might not know is that he was also a mathematician very much interested in logic, and had invented a number of logic puzzles. Here is one of them:
You are given 3 statements.
(a) All babies are illogical.
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are despised.
(d) Can you guess what would be **the natural conclusion** from these 3 statements?

5. Using only *AND*, *NOT*, and *OR*, produce a three-input *AND* circuit, i.e. the output is False unless all three inputs are True. (You do not have to use all three circuit elements.)

6. The diagram on the right shows a circuit constructed from 3 logical chips (each chip has 2 inputs and 1 output; we draw them so that the input lines are on the left and the output line is on the right). Can you determine which input values cause the LED to light up? (Hint: this is the same as writing a truth table for some formula…For example, if A is set to 1 and B is set to 0, what will be the output value of the XOR chip? 1 stands for T, 0 stands for F)



Note that the wires connecting each chip and LED to the power source are not shown.

7. (Optional) Let us consider a new logical operation, called *NAND*, which is defined by the following truth table:

A	*B*	*A NAND B*
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that *A NAND B* is equivalent to *NOT (A AND B)* (this explains the name *NAND* for 'not and')
(b) Show that *A NAND A* is equivalent to *NOT A*
(c) Write the truth table for (*A NAND B) NAND* (*A NAND B)*
(d) Write the truth table for (*A NAND A) NAND* (*B NAND B)*
(e) Show that any logical formula that can be written using *AND*, *OR*, *NOT* can also be written using only *NAND*.

8. On the island of knights and knaves, you meet two inhabitants: X and Y. X says, 'Y is a knave'. Y says 'X is a knave'. Who is a knight and who is a knave?