

Math 6e: Homework 3

HW#3 is Due on October 14; please submit it to Google Classroom 15 minutes before class time.

Please write clearly which problem you are solving and show all steps of your solution.

Classical logic is the study of arguments. An argument is a sequence of statements of which one is a conclusion. The other statements (premises) give evidence or information and help to reach a conclusion. Classical logic evaluates statements as only True or False. Whether a statement is True or False depends on who is evaluating it.

Example of an argument: All humans are mortal. Val is a human. Val is mortal.

Example for evaluating a statement: I am a teacher. **T** (True if I, the teacher, evaluated this statement)
I am a teacher. **F** (False if you, the student, evaluated this statement)

In class, we talked about how to create a simple Truth table to evaluate statements and solve a Knight/Knave problem. These are the guiding steps to follow:

1. Make a table (see the example table attached with class notes, or print and use one of them)
2. Assign all possible combinations of who is a knight/knave for the two people. Write the people's names and "occupations" in the first two columns.
3. Find the statements made by each person in the text; simplify the statements' meaning, if possible, and write the statements in the table in columns 3 and 4.
4. For each row (one combination of Knight/Knave), evaluate the statements yourself based on what you have assigned the people to be.
5. Evaluate the statement from the point of view of the person who made it. If the person is a Knave, they must lie (F); if the person is a Knight, they must tell the Truth (T).
6. For each statement, compare the evaluations from points 4 and 5; if the answers are both the same (TT or FF), this person has been correctly assigned to be a Knight/Knave. If not (answer TF or FT), you have a contradiction – this combination of who the person is assigned to be and the statement made is not logically possible.
7. The row with no contradiction is your solution – read who is the Knave and who is the Knight from it.

Some of the problems in this assignment are from the island of the Knights and Knaves. Remember, Knights always tell the Truth, and Knaves always lie. You can find these problems and many more at:

<http://philosophy.hku.hk/think/logic/knights.php>

Homework questions

1. A hot water faucet fills a bucket in 40 minutes, and the cold water faucet fills the same bucket in 1 hour. How fast will the two faucets fill the bucket when turned on together? (review from math 5)
2. On the island of Knights and Knaves, you meet two inhabitants: Carl and Betty. Carl says, 'Neither Betty nor I are knaves.' Betty claims, 'Carl and I are the same.' Can you determine who is a knight and who is a knave?
3. On the island of Knights and Knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "Both Sue and I are knights." Can you determine who is a knight and who is a knave?
4. On the island of Knights and Knaves, you meet two inhabitants: Bart and Ted. Bart claims, "Ted and I are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave." So who is a knight and who is a knave?

5. **(Optional)** Now imagine that the island also has Normals, who can either tell the Truth or lie. Amy, Bob, and Celine are from the island of Knights, Knaves, and Normals. One of them is a Knight, one is a Knave, and one is a Normal. Amy says that Celine is a Knave. Bob says that Amy is a Knight. Celine says that she is a Normal. Can you figure out who is who?
6. You are in a maze on the island of Knights and Knaves. There are two doors: one leads to freedom, and the other to certain doom. Two guards are nearby, and you know one is a knight and the other a knave, but you don't know which is which. They allow you to ask only one of them a single question before you choose a door — what do you ask?
7. Find the greatest common divisor and least common multiple of 132 and 90.
8. Solve the following equation: $5 - 2(1 - x) = 9$.
9. (Please do not use a calculator to solve the following question.) The maximal distance from the Sun to Pluto is 7,375,927,931 *km*. The speed of light is about 300,000 *km/sec*. How long does it take for the Sun's light to reach Pluto? (You do not need to give a precise answer – an approximate one like “about 2 minutes” would be fine.)