SchoolNova Math 6

Comprehensive Review Problems

Instructions: Show all your work. Your solutions should include explanations of how you obtained your answers. Use algebraic identities and counting principles wherever appropriate, rather than relying on a calculator.

Part A. Algebra and Factorization

1. Simplify and calculate.

(a) Simplify completely:

$$\frac{(2a^3b^2)^2(3ab^3)}{6a^4b^2}.$$

(b) Simplify the radical expression:

$$\sqrt{108} - 2\sqrt{27} + \sqrt{75}.$$

(c) Calculate $103^2 - 97^2$ without directly squaring either number.

2. Algebraic identities.

(a) Expand and simplify:

$$(2x - 3y)^2 - (x + y)(x - y).$$

(b) Calculate $49^3 + 51^3$ using an algebraic identity, without directly cubing either number.

(c) Prove the identity:

$$(a + b)^2 - (a - b)^2 = 4ab.$$

3. Factorize completely.

(a) $x^3 - 4x^2 - 9x + 36$

(b) $16a^4 - 81b^4$

(c) $2x^3 - 8x^2 + 8x$

4. Solve the following equations.

(a) $2x^2 - 7x - 15 = 0$

(b) $x^4 - 13x^2 + 36 = 0$

(c) $(x - 2)(x + 5) = 18$

Part B. Arithmetic Sequences

5. Finding terms of an arithmetic sequence.

An arithmetic sequence has its 4th term equal to 17 and its 11th term equal to 45.

(a) Find the first term a_1 and common difference d.

(b) Find the 25th term.

(c) Which term of the sequence is equal to 101?

6. Arithmetic sequences in real life.

A theater has 15 seats in its first row. Each subsequent row has 3 more seats than the previous row.

(a) How many seats are in the 20th row?

(b) How many seats are there altogether in the first 20 rows?

(c) If the theater has 25 rows, how many additional seats are in rows 21–25?

Part C. Geometric Sequences

7. Finding terms of a geometric sequence.

A geometric sequence has $a_2 = 12$ and $a_5 = 324$. The common ratio is positive.

(a) Find the first term and common ratio.

(b) Find a_7.

(c) Calculate the sum of the first six terms.

8. Geometric series and infinite sums.

(a) Calculate:

$$3 + 6 + 12 + 24 + \cdots + 384.$$

(b) Find the infinite sum:

$$12 + 6 + 3 + \frac{3}{2} + \frac{3}{4} + \cdots .$$

(c) Explain why the infinite sum in part (b) has a finite value, while the corresponding infinite sum in part (a) would not.

Part D. Combinatorics and Permutations

9. Fundamental counting principle and permutations.

A school competition has 8 students.

(a) In how many ways can first, second, and third prizes be awarded if no student can receive more than one prize?

(b) In how many ways can 4 of the 8 students be seated in 4 numbered chairs?

(c) In how many distinct ways can the letters of the word **BALLOON** be arranged? Explain why simply calculating 7! would give an incorrect answer.

10. Combinations and Pascal's triangle.

(a) A teacher wants to select a committee of 3 students from a group of 8 students. How many different committees are possible?

(b) Explain why the answer to part (a) differs from the number of ways to select a president, vice president, and secretary from the same group.

(c) A robot starts at the bottom-left point of a grid and must reach a point exactly 4 steps to the right and 3 steps up. It can only move right or up, one step at a time. How many different paths are possible?

(d) Explain how Pascal's triangle can be used to solve part (c).