Homework 7: Introduction to Combinatorics

HW7 is Due November 15

1. Fundamental Principle of Counting (Multiplication rule).

Combinatorics = counting

If the first task can be performed in **m** ways, and for each of these a second task can be performed in **n** ways, and for each combination a third task can be performed in **k** ways, etc. then this entire sequence of tasks can be performed in **m· n · k** . . . ways.

2. Permutations: *the choice of **k** things from a set of **n** things without repetition("replacement") and where the order matters.*

a) Picking first, second, and third place winners from a group. No replacement – cannot pick the same person twice, order matters – Dave, Emma, Page is a different group than Emma, Dave, Page.
If a group has **n** members, then picking 3 can be done in

$$n\,P_3 = n(n-1)(n-2) \quad \text{ways}$$

b) Permutations of **n** things: The number of permutations of all n different things: n!

- Arranging/ordering all n members of a group can be done in n! ways.
- Listing the favorite deserts in the order of choices: if there are n desserts in total, there are n! ways to arrange them in the order of preference.

3. Combinations: *the choice of **k** things from a set of **n** things without repetition ("replacement") and where **order does not matter**.*

Combinations are harder to count: we will talk about it later and write the formulas!

- Picking three team members from a group of(it doesn't matter who is chosen first, or second, or third).
- Picking two deserts from a tray (the order in which you eat them doesn't matter!).

Picking k-things from n things when order does not matter:

$$n\,C_k = \frac{n!}{(n-k)!k!}$$

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. A dinner in a restaurant consists of 3 courses: appetizer, main course, and dessert. There are 5 possible appetizers, 6 main courses and 3 desserts. How many possible dinners are there?
2. How many ways are there to seat 5 students in a class that has 5 desks? if there are 10 desks?
3. How many ways are there to select first, second and third prize winner if there are 14 athletes in a competition?
4. A dressmaker has two display windows. The left display is for evening dresses and the one in the right window for regular day dresses. Assuming she can put 10 evening dresses in any order, and separately, 5 regular dresses in any order, how many total possibilities of arranging the two display windows are there? [Hint: Use multiplication rule and permutations]

5. The guidelines at a certain college specify that for the introductory English class, the professor may choose one of 3 specified novels, and choose two from a list of 5 specified plays. Thus, the reading list for this introductory class must have one novel and two plays. How many different reading lists could a professor create within these parameters? [Hint: Similar to before]
6. How many unique words can you write from (a) PATTERNS and (b) MISSISSIPPI ? [Hint: Order matters]
7. In how many ways can you choose 5 players from a group of 10 players? [Hint: order does not matter, so need to divide by overcounting factor]