

Homework 3: Algebraic identities, factorization, and Simplification .HW is Due Oct 18th

Basic algebraic identities for refreshing your memory:

Exponents LawsIf a and b are real numbers and n is a positive integer

$$(ab)^n = a^n b^n \quad (\text{eq. 1})$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (\text{eq. 2})$$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (\text{eq.3})$$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad (\text{eq.4})$$

$$\text{Also: } a^2 - b^2 = (a - b)(a + b) \quad (\text{eq. 5})$$

And:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad (\text{eq. 6})$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \quad (\text{eq. 7})$$

Factorization

Just like in arithmetic where you can write large number in terms of the product of its prime factors (for example $30 = 2 \times 3 \times 5$, with 2, 3 and 5 being the prime factors of 30), similarly, algebraic expression can be written as a product of its factors.

In the above expressions, e.g. eq. 5, $(a-b)$ and $(a+b)$ are the factors of $a^2 - b^2$ i.e. $a^2 - b^2 = (a + b)(a - b)$.

Similarly, we can say that $(a+b)$ and $(a+b)$ are the factors of $a^2 + 2ab + b^2$ [In this case, both factors are equal.].

Some examples of factorizing the expressions:

$$1. \quad x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x - 2)(x + 1)$$

$$2. \quad x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 - y^2)(x^2 + y^2) [\text{Applying } (a^2 - b^2) = (a + b)(a - b)] = (x - y)(x + y)(x^2 + y^2)$$

Homework problems

Instructions: Please always write solutions on a **separate sheet of paper**. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Simplify

a) $\frac{1}{a} + \frac{1}{b}$

b) $\frac{3}{x} + \frac{5}{xy} + \frac{5}{3y}$

c) $\frac{x}{(x^2-y^2)} - \frac{y}{(x+y)^2}$

d) $\frac{a}{b} + \frac{b}{c} + \frac{c}{d}$

e) $\frac{a+b}{(b-c)(c-a)} + \frac{b+c}{(c-a)(a-b)} + \frac{c+a}{(a-b)(b-c)}$

2. Factorize i.e write the following as of product of their factors :

a. $x^2 y^2 + 2xy + 1$

b. $xy - by - b + x$

c. $25a^2b^2 - 10abc + c^2$

d. $a^4 - 9b^4$

e. $(x+y)^2 - (x-y)^2$ [Hint: similar to $a^2 - b^2$ form]

f. $(3a-4b)^2 - (5a+6b)^2$

g. $a^4 + 4$ [* unlike above problems, this is addition of squares. Hint: add extra term to write as $(a+b)^2$ and then proceed....]

h. $x^4 + 4y^4$ [similar to problem (g)]