

Homework 1: Algebraic operations

HW1 is Due Sept 27

This week we reviewed basic algebraic rules which are frequently in simplification, factorization and various algebraic problems.

Some basic algebraic rules for you to remember:

1. Exponents Laws

If a is a real number, n is a positive integer

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

2. Radicals

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. Main Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(ax + b)(cx + d) = acx^2 + (bc + ad)x + bd$$

Homework problems

Instructions: Please always write solutions on a **separate sheet of paper**. Solutions should include explanations.

1. Simplify the following radicals:

a. $\sqrt{\frac{56}{13}} \cdot \sqrt{\frac{26}{7}} =$

b. $\sqrt{48} =$

c. $\frac{\sqrt{48}}{\sqrt{15}} =$

2. Rewrite the following expressions in the simplest form, by collecting the like terms, if possible:

a. $2x + 7 + 5x + 3x^2y + yx^2 + 10 =$

b. $2a(a - 2) - a(a - 1) =$

3. Express the following expressions in the form $2^r 3^s a^m b^n$:

a. $8a^3 b^2 (27a^3)(2^5 ab) =$

b. $3^2 (2ab)^3 (16a^2 b^5)(24b^2 a) =$

c. $16a^2 b^3 (6ab^4)(ab^2)^3 =$

4. Expand as sums of powers of x :

a. $(2x + 5)^2 =$

b. $(2 - 4x)^2 =$

c. $(1 - 2x)^2 =$

5. Find the products (use algebraic identities if possible and do not use calculator):

a. $(x^2 + y^2)^2$

e. $(ay+3)(by+5)$

b. $(abc - 1)^2$

f. $(1 - 6y + y^2)^2$

c. 82^2

g. 93×87

d. $(a^2 - 3ax)(a^2 + 3ax)$

h. 993.985

6. Prove:

a. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ [Hint: multiply $(a+b)^2$ by $(a+b)$]

b. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ [similar as above]

c. Find $(a + b)^4, (a - b)^4$ using the previous results