

MATH 6

LOGIC I: KNIGHTS AND KNAVES — SHORT 2026–27

Welcome to a strange island where every inhabitant is one of two kinds: **Knights**, who *always* tell the truth, and **Knaves**, who *always* lie. They look exactly alike. All you can do is listen to what they say — and think. These puzzles were made famous by the mathematician Raymond Smullyan, and they train exactly the kind of careful reasoning mathematicians use every day.

The Rules of the Island

The core rule

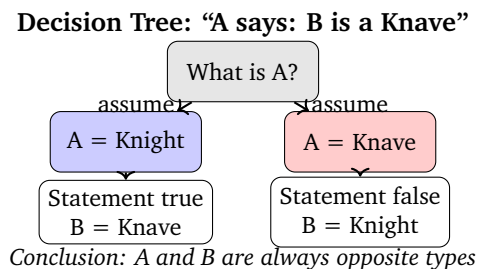
A **Knight's** statements are always true. A **Knave's** statements are always false.

How do we solve a puzzle? We play detective and **try every case**:

1. Suppose the speaker is a Knight. Then the statement must be *true*. Follow the consequences.
2. Suppose the speaker is a Knave. Then the statement must be *false*. Follow the consequences.
3. Throw away every case that leads to a contradiction. Whatever survives is the answer — and there may be one answer, several, or none at all!

Example: the sentence nobody can say. Can anyone say “I am a Knave”? A Knight can’t (it would be a lie), and a Knave can’t either (it would be the truth!). So **no islander can ever say “I am a Knave.”** This little fact cracks many puzzles.

Example: opposite types. A says: “B is a Knave.”



Notice that we never learned *who* is the Knight — only that they are different. Puzzles don’t always reveal everything.

Patterns Worth Remembering

A says...	If A is a Knight	If A is a Knave
“I am a Knight”	true — fine	false — fine
“I am a Knave”	false — impossible!	true — impossible!
“B is a Knight”	B is a Knight	B is a Knave
“B is a Knave”	B is a Knave	B is a Knight
“We are the same type”	same type, so B is a Knight	really different, so B is a Knight
“We are different types”	really different, so B is a Knave	really the same, so B is a Knave

Look at the last two rows: whatever A is, “We are the same type” tells us that B is a Knight, and “We are different types” tells us that B is a Knave!

Two People Talking

Example. Alice says: “We are the same kind.” Bob says: “We are of different kinds.”

The two statements are exact opposites, so exactly one of them is true. That means one speaker is a Knight and the other a Knave — so they *are* of different kinds. Bob’s statement is the true one: **Bob is the Knight, Alice is the Knave.**

Example: three people. A says: “C is a Knave.” B says: “A is a Knight.” C says: “I am a Knight.”

Case 1: B is a Knight. Then A is a Knight, so C is a Knave — and a Knave may falsely say “I am a Knight.” ✓

Case 2: B is a Knave. Then A is a Knave, so C is a Knight — and a Knight truly says “I am a Knight.” ✓

Both cases work, so this puzzle has **two** solutions. That’s a perfectly good answer: always report *every* assignment that works.

Asking the Right Question

Sometimes *you* get to ask. The problem: you don’t know whether you’re talking to a Knight or a Knave.

The “would you say” trick

Ask: “*If I asked you whether . . . , would you say yes?*”

A Knight answers honestly. A Knave lies about the fact, and then lies *again* about what he would say — the two lies cancel! So **both** types answer truthfully.

Example. At a fork in the road, one path leads to Freedom and one to Death; one guard stands there. Ask him: “If I asked you whether the left road leads to Freedom, would you say yes?” A “yes” means the left road leads to Freedom, whoever he is.

A second classic trick, for two guards (one Knight, one Knave): ask either one what *the other* would say. The Knight faithfully reports the Knave’s lie; the Knave lies about the Knight’s truth. Either way you hear the *wrong* answer — so do the opposite!

Normals and the Island of Bahava

Some puzzles add a third kind of person: a **Normal**, who sometimes lies and sometimes tells the truth. A Normal can say anything at all, so a Normal’s statement tells us nothing by itself.

On the imaginary island of **Bahava**, there is an extra marriage rule: a Knight may marry only a Knave, a Knave only a Knight, and a Normal only another Normal.

Example. A married couple from Bahava, Mr. and Mrs. A, both say: “My spouse is not normal.”

If Mr. A were a Knave, his wife would be a Knight — so “my wife is not normal” would be *true*, which a Knave can’t say. So Mr. A is not a Knave, and by the same argument neither is Mrs. A. They can’t be Knights either (a Knight’s spouse is a Knave). **So both are Normal** — and both are lying!

Common Mistakes

- **Checking only one case.** Always test *both* possibilities for each speaker.
- **Stopping at the first answer.** Some puzzles have several solutions — and some have none.
- **Forgetting the “I am a Knave” rule.** Nobody on the island can say it.
- **Treating Normals like Knaves.** A Normal *can* tell the truth; their statements alone prove nothing.

Homework

1. Alice and Bob are standing next to each other. Alice says, “At least one of us is a Knight.” Bob says, “Alice is a Knave.” What are Alice and Bob?
2. You meet two inhabitants: Claire and Desmond. Claire suddenly says, “We are both Knights.” After this, Desmond says “Either Claire is a Knight or I am a Knight, but we are not both Knights.” What are Claire and Desmond?
3. **H** There are three native islanders, named Elena, Fernando, and Gary, standing together. You ask Elena, “How many Knights are among you?” Elena answers, but you cannot quite hear what she says. You then ask Fernando, “What did Elena say?” Fernando replies, “Elena said there is one Knight among us.” At this point Gary says, “Don’t believe Fernando; he is lying.”
Is it possible to determine what Elena is? Is it possible to determine what Fernando is? Is it possible to determine what Gary is?
4. You meet two inhabitants: Alice and Bob. Alice tells you that Bob is a knave. Bob says, “Neither Alice nor I are knaves.” Who is a knight and who is a knave?
5. Three inhabitants—A, B, and C—are standing together in a garden. A traveler asks A, “Are you a Knight or a Knave?” A answers too quietly to be heard. The traveler then asks B, “What did A say?” B replies, “A said that he is a Knave.” C says, “Don’t believe B; he is lying!” What are B and C?
6. Suppose Bridget says, “Either I am a knave or Connor is a knight.” What are Bridget and Connor?
7. Harry says that all three people—Harry, Irma, and Janet—are Knaves. Irma then says that there is exactly one Knave among them. What can you conclude?
8. Suppose Deb says, “I am a knave, but Ed isn’t.” What are Deb and Ed?
9. We again have three inhabitants, Alice, Bob, and Charlie, each of whom is a knight or a knave. Two people are said to be *of the same type* if they are both knights or both knaves. Alice and Bob make the following statements:
Alice: “Bob is a knave.”
Bob: “Alice and Charlie are of the same type.”
What is Charlie?
10. **M** Harry, Irma, and Jane are standing together. Harry says, “Irma and Jane are of the same type.” Someone then asks Jane, “Are Harry and Irma of the same type?” What does Jane answer?
11. Consider a married couple on the Island of Bahava, Mr. and Mrs. A. They make the following statements:
Mr. A: My wife is normal.
Mrs. A: My husband is normal.
What are Mr. and Mrs. A?
12. **M** **Who are you?** You meet a single islander and want to find out whether he is a Knight or a Knave. The direct questions “Are you a Knight?” and “Are you a Knave?” don’t work — why not? Find one yes/no question that does.
13. A says: “B is a Knight.” B says: “A is a Knave.” What are A and B?