

MATH 6: HANDOUT 01

REVIEW OF MATH 5 — SHORT 2026–27

Fractions, Decimals, and Percents

Every denominator must be nonzero. To add or subtract fractions, first use a common denominator:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}.$$

Multiply numerators and denominators; to divide, multiply by the reciprocal of the second fraction. Reduce a fraction by dividing its numerator and denominator by their greatest common divisor (GCD). A terminating decimal is a fraction whose denominator can be written as a power of 10; convert a fraction to a decimal by dividing. A percent is a rate per hundred: $p\% = \frac{p}{100}$, so $p\%$ of N is $\frac{p}{100} \cdot N$.

Negative Numbers and Absolute Value

Subtracting means adding the opposite: $a - b = a + (-b)$. When adding numbers with the same sign, add their magnitudes and keep the sign; with different signs, subtract magnitudes and keep the sign of the number with greater magnitude. Products and quotients with equal signs are positive; with different signs, they are negative. Absolute value is distance from zero: $|x| \geq 0$, and $|a - b|$ is the distance between a and b . Thus $|x| = a$ has solutions $x = \pm a$ when $a > 0$.

Ratios and Proportions

A ratio compares quantities; convert to compatible units before comparing and simplify like a fraction. Equivalent ratios are made by multiplying or dividing both terms by the same nonzero number. A unit rate tells how much per one unit. A proportion is an equality of ratios; if $b, d \neq 0$, then

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc.$$

Use unit rates and proportions to scale recipes, maps, prices, and similar figures. Keep track of units so the answer matches the question.

Exponents and Roots

For integer exponents and nonzero bases where division occurs,

$$a^m a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}, \quad (ab)^n = a^n b^n,$$

with $a^0 = 1$ and $a^{-n} = 1/a^n$. A square root is the nonnegative number whose square is the given value, so $\sqrt{x^2} = |x|$. For nonnegative a, b , $\sqrt{ab} = \sqrt{a}\sqrt{b}$; factor out perfect squares to simplify (for example, $\sqrt{50} = 5\sqrt{2}$). The square root of a nonnegative integer that is not a perfect square is irrational.

Expressions and Equations

Combine like terms only when their variable parts match; distribute by multiplying every term inside parentheses: $a(b + c) = ab + ac$. To solve a linear equation, keep it balanced by doing the same operation to both sides, undoing operations until the unknown is isolated. For a word problem, name the unknown, translate the relationships into an equation, solve, and check that the answer makes sense in context.

Inequalities

Solve an inequality much like an equation, but multiplying or dividing both sides by a negative number reverses the inequality sign. Graph solutions on a number line with an open circle for $<$ or $>$ and a closed circle for $\leq$ or $\geq$. For compound inequalities, “and” means both conditions must hold (intersection); “or” means at least one holds (union).

Geometry Essentials

Complementary angles sum to 90° ; supplementary angles sum to 180° ; vertical angles are equal. Triangle angles sum to 180° ; a triangle is acute if all its angles are less than 90° , right if one is 90° , and obtuse if one is greater than 90° . Useful area formulas are

$$A_{\triangle} = \frac{1}{2}bh, \quad A_{\text{rect}} = bh, \quad A_{\text{para}} = bh, \quad A_{\text{trap}} = \frac{1}{2}(b_1 + b_2)h, \quad A_{\text{circle}} = \pi r^2,$$

and a circle’s circumference is $C = 2\pi r$. Use square units for area and linear units for circumference. In a right triangle, the legs a, b and hypotenuse c satisfy $a^2 + b^2 = c^2$. Similar triangles have equal corresponding angles and proportional corresponding sides.

Counting, Probability, and Sets

For equally likely outcomes, probability is between 0 and 1 and

$$P(A) = \frac{\text{favorable outcomes}}{\text{total outcomes}}, \quad P(\text{not } A) = 1 - P(A).$$

Multiply the numbers of choices in successive stages; add counts for nonoverlapping cases. For independent events, $P(A \text{ and } B) = P(A)P(B)$. There are $n! = n(n-1) \cdots 2 \cdot 1$ orders of n distinct objects, and $0! = 1$. For sets, $\cup$ means “or,” $\cap$ means “and,” and

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

If n objects are placed in k boxes with $n > k$, some box contains at least two objects (the pigeonhole principle).

Common Mistakes to Avoid

- A later percent change applies to the new amount, not the original one; equal increases and decreases do not generally cancel.
- In probability, count favorable and total outcomes using the same model: both ordered or both unordered.
- In the Pythagorean theorem, c is the hypotenuse, opposite the right angle. Area uses square units; perimeter and circumference use linear units.
- Keep restrictions and units visible: denominators cannot be zero, and quantities in a ratio must use compatible units.

Homework

Difficulty: Unmarked problems are routine review; **M** marks a moderate problem; **H** marks a hard or extra-credit problem.

1. Write 0.45 as a fraction in lowest terms, and write $\frac{3}{8}$ as a decimal.
2. Evaluate $(-4)(5) - (-3)(2)$.
3. A car travels 150 miles on 5 gallons of gas. At the same rate, how far can it travel on 8 gallons?
4. Evaluate $3^{-2} + 5^0$.
5. Solve $4(x - 2) = 2x + 6$.
6. Solve $|x| = 7$.
7. Two angles of a triangle are equal, and the third is 80° . Find all three angles.
8. A 15-foot ladder leans against a wall with its base 9 feet from the wall. How high up the wall does the ladder reach?
9. **M** Two fair dice are rolled. What is the probability that the sum is 7?
10. **H** In how many ways can 5 people sit in a row? In how many of those ways do two particular people, Alice and Bob, sit next to each other?
11. **M** A number is increased by 20% and then decreased by 20%. The final value is 96. Find the original number. Explain why the two changes do not cancel.
12. A jacket is discounted by 20%, and then 8% sales tax is added to the discounted price. The total paid is 86.40. What was the original price, and how much tax was paid?