MATH 6

REVIEW OF MATH 5 — SHORT 2026–27

Welcome back! Last year in Math 5 you built a big toolkit: fractions and decimals, negative numbers, ratios, exponents and roots, algebra, inequalities, geometry, and the beginnings of counting and probability. Here is the whole toolkit on a few pages — for each topic, the rules in a box and one worked example. Keep it handy: we will use all of it this year.

Fractions, Decimals, and Percents

Refresher

- In every fraction, the denominator must be nonzero.
- **Add/subtract:** put over a common denominator, then combine: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$, where $b, d \neq 0$.
- **Multiply:** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. **Divide:** multiply by the reciprocal, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b, c, d \neq 0$.
- **Simplify** by dividing top and bottom by their GCD.
- **Decimals:** a terminating decimal is a fraction over a power of 10 (e.g. $0.7 = \frac{7}{10}$); a fraction becomes a decimal by dividing.
- **Percent:** $x\% = \frac{x}{100}$. "$p\%$ of N" means $\frac{p}{100} \cdot N$.

Example 1. Compute $\frac{5}{6} - \frac{3}{8}$, write 0.24 as a fraction in lowest terms, and find 15% of 80.

Solution. Common denominator 24: $\frac{5}{6} - \frac{3}{8} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$.
$0.24 = \frac{24}{100} = \frac{6}{25}$.
15% of $80 = \frac{15}{100} \cdot 80 = 12$.

Negative Numbers and Absolute Value

Refresher

- **Subtracting** is adding the opposite: $a - b = a + (-b)$.
- **Signs in $\times$ and $\div$:** same signs give a *positive* result, different signs give a *negative* one. (For example $(-3)(-4) = 12$ and $(-3)(4) = -12$.)
- **Absolute value** $|x|$ is the distance from x to 0, so $|x| \geq 0$ always.
- $|a - b|$ is the distance between a and b on the number line.

Example 2. Compute $(-3) - (-7) + (-4) \cdot 2$ and $|-5| + |3 - 9|$.

Solution. $(-3) - (-7) = (-3) + 7 = 4$, and $(-4) \cdot 2 = -8$, so $4 + (-8) = -4$.
$|-5| + |3 - 9| = 5 + |-6| = 5 + 6 = 11$.

Ratios and Proportions

Refresher

- A **ratio** $a : b$ compares two quantities; express measured quantities in compatible units, then simplify the ratio like a fraction.
- A **proportion** is an equality of ratios: $\frac{a}{b} = \frac{c}{d}$. If $b, d \neq 0$, **cross-multiply** to get $ad = bc$.
- A **unit rate** is "how much per one" (e.g. miles per gallon, cost per item).
- Proportions scale recipes, maps, and similar shapes.

Example 3. A recipe uses 3 cups of flour for 12 cookies. How much flour is needed for 20 cookies? Also solve $\frac{4}{5} = \frac{x}{35}$.

Solution. $\frac{3}{12} = \frac{x}{20} \Rightarrow 12x = 60 \Rightarrow x = 5$ cups.
$\frac{4}{5} = \frac{x}{35} \Rightarrow 5x = 140 \Rightarrow x = 28$.

Exponents and Roots

Refresher

- $a^m \cdot a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$ for $a \neq 0$, $(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$.
- $a^0 = 1$ (for $a \neq 0$), $a^{-n} = \frac{1}{a^n}$ (for $a \neq 0$).
- For real numbers, $\sqrt{x^2} = |x|$. If $a, b \geq 0$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so pull out perfect squares (e.g. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$).
- If a nonnegative integer a is not a perfect square, then $\sqrt{a}$ is **irrational** (like $\sqrt{2}$): it cannot be written as a fraction.

Example 4. Simplify $2^3 \cdot 2^4$, $(3^2)^2$, and $\sqrt{72}$.

Solution. $2^3 \cdot 2^4 = 2^7 = 128$.
$(3^2)^2 = 3^4 = 81$.
$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$.

Algebraic Expressions and Equations

Refresher

- **Combine like terms** (same variable part): $3x + 5x = 8x$.
- **Distribute:** $a(b + c) = ab + ac$.
- **Solve a linear equation** by undoing operations, doing the *same thing to both sides*, until x is alone.
- **Word problems:** name the unknown, translate the words into an equation, solve, and check.

Example 5. Simplify $3(x+2)-2(x-1)$, then solve $5x-7=2x+8$.

Solution. $3(x+2)-2(x-1)=3x+6-2x+2=x+8$.
$5x-7=2x+8 \Rightarrow 3x=15 \Rightarrow x=5$.

Inequalities

Refresher

- Solve like an equation, with **one rule change**: multiplying or dividing both sides by a *negative* number **flips** the inequality sign.
- **Graph** on a number line: open circle ($\circ$) for $<,>$; closed circle ($\bullet$) for $\leq,\geq$.
- **Compound:** "AND" means satisfying both inequalities—the intersection of their solution sets. "OR" means satisfying at least one—the union. In common two-bound problems, AND gives values between two bounds ($2<x\leq 7$), while OR can give values outside them ($x<1$ or $x>5$).

Example 6. Solve $3x-5<7$ and solve $-2x\geq 6$.

Solution. $3x-5<7 \Rightarrow 3x<12 \Rightarrow x<4$.
$-2x\geq 6 \Rightarrow x\leq -3$ (divide by -2, **flip**).

Geometry Essentials

Refresher

- **Angles:** a right angle is 90°, a straight angle is 180°. Complementary angles sum to 90°; supplementary angles sum to 180°; *vertical* angles (across an X) are equal.
- **Triangle angle sum** $=180^\circ$. A triangle is acute (all $<90^\circ$), right (one $=90^\circ$), or obtuse (one $>90^\circ$).
- **Areas:** rectangle $=bh$; triangle $=\frac{1}{2}bh$; parallelogram $=bh$; trapezoid $=\frac{1}{2}(b_1+b_2)h$; circle $=\pi r^2$.
- **Circle:** circumference $=2\pi r$.
- **Pythagorean theorem** (right triangle, legs a,b, hypotenuse c): $a^2+b^2=c^2$.
- **Similar triangles** have equal corresponding angles and *proportional* corresponding sides.

Example 7. A triangle has angles 50° and 60°. Find the third angle. Then find the area of a triangle with base 8 and height 5, and the area of a circle of radius 3.

Solution. Third angle $=180^\circ-50^\circ-60^\circ=70^\circ$.
Triangle area $=\frac{1}{2}(8)(5)=20$.
Circle area $=\pi(3)^2=9\pi\approx 28.3$.

Counting, Probability, and Sets

Refresher

- **Probability** (equally likely outcomes) $= \dfrac{\text{favorable}}{\text{total}}$, a number between 0 and 1.
  Complement: $P(\text{not } A) = 1 - P(A)$.
- **Counting multiplication principle:** if one choice has m options and each of those choices allows n options for the next choice, together there are $m \cdot n$ possibilities.
- **Probability multiplication rule:** for independent events, $P(A \text{ and } B) = P(A) \cdot P(B)$.
- **Arrangements:** n different objects can be ordered in $n! = n \cdot (n-1) \cdots 2 \cdot 1$ ways.
- **Sets:** $\cup$ is "or", $\cap$ is "and". **Inclusion–exclusion:** $|A \cup B| = |A| + |B| - |A \cap B|$.
- **Pigeonhole:** if n items go into k boxes and $n > k$, some box holds at least two.

Example 8. Roll a fair die: what is $P(\text{even})$? Flip two coins: what is $P(\text{two heads})$? In a class of 30, 18 play soccer, 15 play basketball, and 8 play both — how many play at least one?

Solution. $P(\text{even}) = \dfrac{3}{6} = \dfrac{1}{2}$.
$P(\text{two heads}) = \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}$.
At least one $= 18 + 15 - 8 = 25$ students.

Key Takeaways

- Fractions: common denominator to add; multiply straight across; divide by flipping.
- Signs: same signs multiply to $+$, different to $-$; $|x|$ is a distance (≥ 0).
- Proportions: cross-multiply. Exponents: add powers to multiply, subtract to divide.
- Equations: same operation to both sides. Inequalities: same idea, but *flip* on a negative multiply/divide.
- Triangle angles sum to $180°$; circle area $= \pi r^2$, circumference $= 2\pi r$; right triangles obey $a^2 + b^2 = c^2$.
- Probability $= \frac{\text{favorable}}{\text{total}}$; independent events multiply; $|A \cup B| = |A| + |B| - |A \cap B|$.

Homework

1. Write 0.45 as a fraction in lowest terms, and write $\dfrac{3}{8}$ as a decimal.
2. Evaluate $(-4)(5) - (-3)(2)$.
3. A car travels 150 miles on 5 gallons of gas. At the same rate, how far can it travel on 8 gallons?
4. Evaluate $3^{-2} + 5^{0}$.
5. Solve $4(x - 2) = 2x + 6$.
6. Solve $|x| = 7$.
7. Two angles of a triangle are equal, and the third is 80°. Find all three angles.
8. A 15-foot ladder leans against a wall with its base 9 feet from the wall. How high up the wall does the ladder reach?
9. M Two fair dice are rolled. What is the probability that the sum is 7?
10. H In how many ways can 5 people sit in a row? In how many of those ways do two particular people, Alice and Bob, sit next to each other?
11. M A number is increased by 20% and then decreased by 20%. The final value is 96. Find the original number. Explain why the two changes do not cancel.
12. A jacket is discounted by 20%, and then 8% sales tax is added to the discounted price. The total paid is 86.40. What was the original price, and how much tax was paid?