

MATH 6

HANDOUT 5: LOGIC IV: BOOLEAN ALGEBRA AND COMPUTERS

BASIC LOGIC OPERATIONS

For your convenience, here is the list of logic operations we have seen so far:

- NOT A : true if A is false, false if A is true
- A AND B : true if both A and B are true, false otherwise
- A OR B : true if at least one of A and B is true, false otherwise
- A XOR B : true if exactly one of A and B is true, false otherwise
- A NAND $B = \text{NOT}(A \text{ AND } B)$: false if both A and B are true, true otherwise
- $A = B$: true if A and B are both false or both true, false otherwise
- $A \neq B$: equivalent to $A \text{ XOR } B$

IF

Recall that we are studying logic rules, in particular logic rules involving operation $\Rightarrow$ (reads “implies”, or “if A then B ”). Here are some of the more important rules:

- Statement $A \Rightarrow B$ is false when A is true and B is false; in all other situations $A \Rightarrow B$ is true.
- $A \Rightarrow B$ and $B \Rightarrow A$ are not equivalent: it is possible that one statement is true and the other is false.
- Contrapositive rule: $A \Rightarrow B$ is equivalent to $(\text{NOT } B) \Rightarrow (\text{NOT } A)$.
- $\text{NOT}(A \Rightarrow B)$ is equivalent to $A \text{ AND } (\text{NOT } B)$

This construction is very useful in deducing new results from known ones. Here are some of the rules:

- Given $A \Rightarrow B$ and $B \Rightarrow C$, we can conclude $A \Rightarrow C$
- Given $A \Rightarrow B$ and $\text{NOT } B$, we can conclude $\text{NOT } A$

LOGIC AND COMPUTER CHIPS

In 1854, George Boole, an English mathematician worked on an algebraic system in which logic variables use 0 and 1. True stands for 1 and False stands for 0. We call it Boolean Algebra.

In 1937, Claude Shannon wrote his master’s thesis at MIT, in which he proposed using Boole’s binary logic to control relay and switching circuits. This was a brilliant idea that revolutionized computer hardware engineering. His work led to developing logic gates. Today, microprocessors contain millions of logic gates that are capable of executing binary arithmetic.

We have seen in class that there are computer chips that can reproduce logical operations: each of the inputs and outputs can have voltage 0 or a positive voltage. The usual convention is

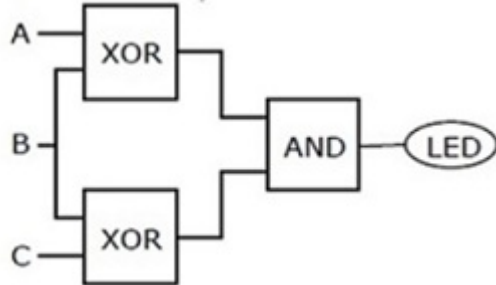
- Positive voltage=true
- Zero voltage=false

Then one can relatively easily construct AND, NOT, ... chips, and combining them, more complicated chips.

HOMEWORK

1. Many trucks carry the message: “If you do not see my mirrors, then I do not see you”. Can you rewrite it in an equivalent form without using the word “not”?
2. If today is Thursday, then Jane’s class has library day. If Jane’s class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore, ...
3. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, ...
4. Check that $\text{NOT}(A \Rightarrow B)$ is equivalent to $A \text{ AND } (\text{NOT } B)$.
What is the equivalent of saying “It is not true that if an animal is a bird, it can fly”?
5. Are formulas $A \Rightarrow B$ and $B \Rightarrow A$ equivalent? Are formulas $A \Rightarrow B$ and $\text{NOT } A \Rightarrow \text{NOT } B$ equivalent? Check this by writing the truth tables for each of them.

6. A mom tells her son “If you do not do the dishes, you will not go to the movie”. Is it the same as “If you do the dishes, you go to the movie?”
7. The diagram below shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? [Hint: this is the same as writing a truth table for some formula....]



Note: the wires connecting each of the chips and LED to the power source are not shown.

8. *(a) On the island of knights and knaves, you meet two inhabitants, X and Y. X says, “Y is a knave”. Y says, “X is a knave”. Who is a knave and who is a knight?
- *(b) Consider the circuit below: there are two NAND chips, and output of each one is connected to the input of the other. If inputs A,B are both true (i.e., carry positive voltage), what will be the values of X,Y?

