Math 5e, Fall 2025 Homework 9

Homework #8 is due November 23

Instructions: Some of the problems were solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

General notation (n is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \ (n \text{ times})$$

Special cases:

$a^0 = 1$	read: a-to-the-zero
$a^1 = a$	is just itself 'a'
$a^2 = a \times a$	read: a-squared
$a^3 = a \times a \times a$	read: a-cubed

1. Solve the following equations:

   (a) $5(x-1) - 4 = 3x + 1$
   (b) $\frac{2}{3}(x-2) = -18$
   (c) $|2x + 1| = 7$

2. When Dennis was 27, his son was three years old. Now, his son's age is one-third of Dennis' age. How old is each of them now?

3. Find the sum of $1 + 2 + 4 + \cdots + 2^n$ (the answer will depend on n). [Hint: first try computing it for several small values of n, for example $n = 4$ is $1 + 2 + 3 + 4 = 2^0 + 2^1 + 2^3 + 2^4$, see if a pattern emerges, then find a general rule.]

4. If we put one grain of wheat on the first square of the chessboard, two on the second, then four, eight, … approximately how many wheat grains will there be? (You can use $2^{10} = 1024 \approx 1000$.)

5. Lotus flowers are growing in a lake. Every day, each lotus plant divides into two plants, so the area its leaves cover is doubled. In 30 days the whole lake is covered with lotus leaves. When did leaves cover exactly half of the lake?

6. Evaluate carefully (watch signs and parentheses)

   (a) $(2.3)^3$ (b) $2 \cdot 3^3$ (c) $\left(\frac{1}{3}\right)^2$ (d) $\frac{1}{3^2}$

7. Compare (use $>, <, =$)

   (a) 2^5 and 5^2
   (b) 3^4 and 4^3
   (c) 2^2 and $2 \cdot 2$
   (d) 2^4 and 4^2

8. Evaluate the following expressions. Hint: first present as a fraction with a denominator of 10^n
   (a) $Example:\ 0.2^6 = \left(\frac{2}{10}\right)^6 = \frac{2^6}{10^6} = 2^6 \cdot 10^{-6} = 64 \cdot 10^{-6} = 0.000064$
   (b) $0.05^2;\ (-0.05)^2;\ -0.05^2$
   (c) $10^3;\ (-10)^3;\ -10^3;\ 10^{-3}$
   (d) $0.1^3;\ (-0.1)^3;\ -0.1^3;\ 0.1^{-3}$

9. Write as powers with base 2:
   (a) $Example: 64 \times 2^2 = 2^6 \times 2^2 = 2^{6+2} = 2^8$
   (b) 32×2^3.
   (c) $2^5 \times 2^3 \times 16$
   (d) $2^3(16 + 2^4)$

10. Simplify the expressions and write them as products of numbers in base with a power form
   (a) $\frac{2^5 \cdot 3^4}{3^2 \cdot 2^3}$
   (b) $\frac{(-2)^9 \cdot 3^8}{2^3 \cdot (-9)^4}$
   (c) $\frac{a^5 \cdot b^4}{b^2 \cdot a^2 \cdot a}$
   (d) $3xy^2(xy^3)^3$

11. Find the prime factorization of 500 and 1215. Express them as products of powers and find the greatest common factor (GCF).