

MATH 5: HANDOUT 5

ALGEBRAIC EXPRESSIONS AND FRACTIONS

Division and Fractions

So far we have studied the rules of addition, subtraction, and multiplication. Now we complete the picture with **division and fractions**.

The key idea is that division can always be written as a fraction:

$$a \div b = \frac{a}{b}.$$

Rules for Fractions

For any nonzero numbers b, d :

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

When the numerator has addition or subtraction:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}, \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}.$$

Examples

1. Simplify $a \div (b \div c)$:

$$a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}.$$

2. Solve $\frac{5}{7}x = 15$. Multiply both sides by $\frac{7}{5}$:

$$x = 15 \times \frac{7}{5} = \frac{105}{5} = 21.$$

3. Compute $\frac{2}{3} \div \frac{5}{6}$:

$$\frac{2}{3} \times \frac{6}{5} = \frac{12}{15} = \frac{4}{5}.$$

Common Mistakes to Avoid

- Remember that $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$.
- Division by zero is undefined.
- Always reduce fractions when possible.

Homework

1. Compute:

(a) $5 \div 7\frac{1}{2}$

(b) $(2 - 3.5) \times \frac{2}{3}$

(c) $3\frac{2}{11} \div \frac{5}{7}$

2. Simplify each expression.

(a) $x + 4(1 - x)$

(b) $2 + 5x - 4(3 - x)$

(c) $5(x - 1) - 3(2x + 1)$

(d) $x - \frac{4}{5}(1 - x)$

(e) $\left(\frac{2}{3}x + 1\right) \div \frac{6}{7}$

(f) $2 \div (3 \div x - 1)$

3. Solve the equations.

(a) $\frac{3}{4}x = 12$

(b) $\frac{1}{2}x + \frac{1}{7}x = 18$

(c) $\frac{2}{3}(x + 7) = 12$

(d) $2.1x + \frac{1}{5}x + 1 = 9$

4. John bought a large bag of red, green, and blue candies, 74 candies in all. The number of red candies is one less than the number of green candies, and there are as many blue candies as red and green together. How many candies of each color are there?
5. A boy had a bag of apples. He gave $\frac{1}{2}$ of them to his parents, $\frac{1}{5}$ to his brother, $\frac{1}{4}$ to his sister, and ate the last apple himself. How many apples did he originally have?
6. Jane is 5 years old and her father is 42. In how many years will he be twice as old as she? Three times as old? (Hint: denote the number of years by x and write equations for x .)
7. A hot water tap fills the bath in 5 minutes. The cold water tap fills the bath in 3 minutes. With both taps open, how long will it take to fill the bath?
8. A watermelon is 99% water (by weight). Watermelon jelly is 98% water. How much water must evaporate from 1 kg of watermelon to turn it into jelly?
9. Using five 3s, arithmetic operations, and parentheses, make five expressions that equal to numbers from 6 to 10.
10. We'll call a number "lucky" if, between its digits, we can insert arithmetic operation signs (+, −, ×, ÷) and parentheses so that the resulting expression equals 100. Is number 123456 lucky?
- *11. Forty children were holding hands in a circle. Among them, 22 were holding a boy's hand and 30 were holding a girl's hand. How many girls were there in the circle?
- *12. The numbers 1, 2, ..., 10 are written in a row. Is it possible to put signs + and − between them so that the result is zero?