Math 5b. Class work 4.

Scientific notation.

Scientists study objects that are very large and very small, from galaxies to viruses. They need to be able to write numbers, describing the object of interest, for example the distance between two galaxies or the diameter of a virus.

One of the most important numbers in the universe is the speed of light.

299 792 458 m / s. It's very convenient to represent it as a decimal between 1 and 10 and multiplied by a power of 10.

$$299\ 792\ 458\ m\ per\ s = 2.99792458 \cdot 10^8\ m\ p\ s.$$

Since 10^8 is a vary large number, it's safe to say, that the speed of light is very close to 300000000 m p s.

$$299\ 792\ 458\ m\ per\ s \approx 300000000\ m\ per\ s = 3 \cdot 10^8$$

Such representation is called scientific notation.

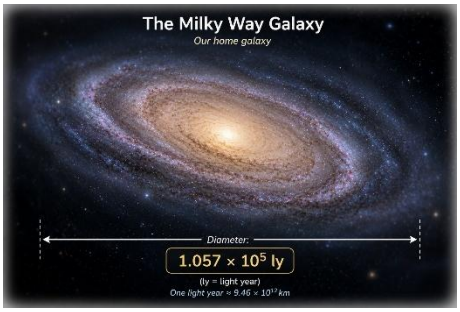
Let's convert the value to kilometers per hour. Each kilometer is 1000 meters, so we need to divide it by 1000:

$$3 \cdot \frac{10^8}{10^3} kmps = 3 \cdot 10^{8-5} = 3 \cdot 10^5 km\ per\ s$$

There are 3600 seconds in one hour, or $3.6 \cdot 10^3$ seconds. To find out the speed of light in km per hour we now need to multiply the speed in seconds by $3.6 \cdot 10^3$

$$3 \cdot \frac{10^8}{10^3} km\ p\ s = 3 \cdot 10^{8-5} = 3 \cdot 10^5 km\ per\ s = 3 \cdot 10^5 \cdot 3.6 \cdot 10^3 km\ p\ h = 10.8 \cdot 10^8\ km\ p\ h$$

$$= 1.08 \cdot 10 \cdot 10^8 = 1.08 \cdot 10^9 km\ p\ h$$



The Milky Way galaxy has a diameter of 105,700 light years, so the light will travel from one end to the other through its center in 105700 years.

How far is it from one side of the Milky Way to the other in kilometers?

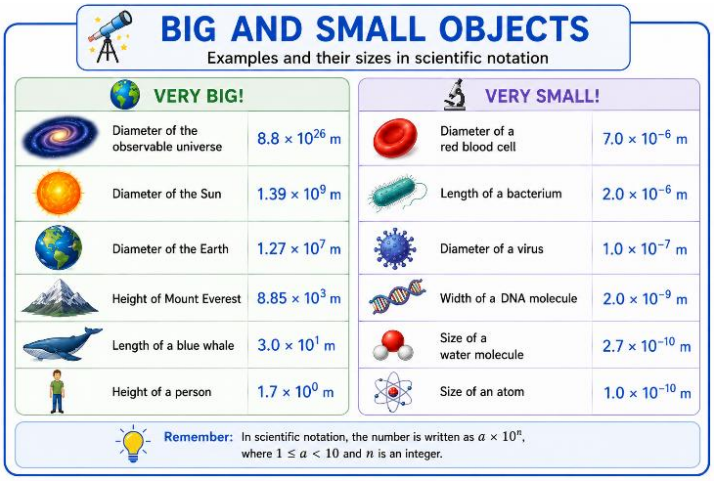
$$1.08 \cdot 10^9 km\ p \cdot 105700\ years$$

How many hours in a year? $24 \cdot 365.25 = 8766 \approx 8.8 \cdot 10^3\ hours$

$$1.08 \cdot 10^9\ km\ p\ h\ \cdot 105700\ years \approx 1.08 \cdot 10^9\ km\ p\ h \cdot 8.8 \cdot 10^3 \cdot 1.06 \cdot 10^5 \approx 10.07 \cdot 10^{17} km$$
$$\approx 10^{18} km.$$

This way to write numbers is called scientific notation, it's used a lot in science for describing various objects, big and small. The number should be represented as a product of one-digit whole-number part with decimal part rounded to two decimal places and a power of 10. For example, the distance from Earh to the Moon is about 238,855 miles (384,400 kilometers).

$$238855 = 2.38855 \cdot 10^5 \approx 2.39 \cdot 10^5\ miles$$



Let's take a look at some very small objects, such as bacteria and viruses.

$$1cm = 0.01m = \frac{1}{100}m = \frac{1}{10^2}m = 10^{-2}m$$

$$1mm = 0.001m = \frac{1}{1000}m = \frac{1}{10^3}m = 10^{-3}m$$

$$1\mu m = 10^{-6}m, \quad 1nm = 10^{-9}m$$

Bacteria are between $0.5 - 1.5\ \mu m$

$$0.5\mu m = 0.5 \cdot 10^{-6}m = 5 \cdot 10^7 m$$

Viruses can be as small as $20\ nm$ ($2 \cdot 10 \cdot 10^{-9}m = 2 \cdot 10^{-8}m$) and as large (in the virus world, of course) as $300\ nm$ ($3 \cdot 10^2 \cdot 10^{-9}m = 3 \cdot 10^{-7}m$)

Problem solving examples:

Example 1:

Prove that

$8^5 + 2^{11}$ is divisible by 17

$$8^5 = (2 \cdot 2 \cdot 2)^5 = (2^3)^5 = 2^{15}$$

$$2^{15} = 2^{11+4} = 2^{11} \cdot 2^4$$

Therefore,

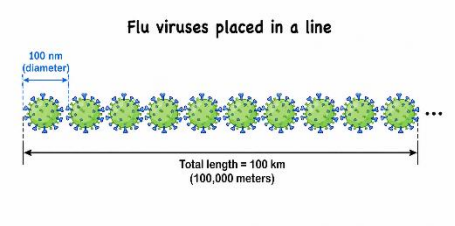
$$8^5 + 2^{11} = 2^{11} \cdot 2^4 + 2^{11}$$

2^{11} is a common factor and can be factor out:

$$8^5 + 2^{11} = 2^{11} \cdot 2^4 + 2^{11} = 2^{11} \cdot (2^4 + 1) = 2^{11} \cdot (16 + 7) = 2^{11} \cdot 17$$

So, 17 is a factor of the value of the expression $8^5 + 2^{11}$, and this value is divisible by 17.

Example 2:



A flu virus is about 100 nm in diameter. During a typical flu infection, an infected person is estimated to produce about 10^{12} virus particles. If all of these virus particles were arranged in a straight line, touching one another, how long would the line be? Express your answer in kilometers.

$$100\ nm = 100 \cdot 10^{-9} m = 10^2 \cdot 10^{-9} m = 10^{2-9} m = 10^{-7} m$$

$$10^{-7} m \cdot 10^{12} = 10^{-7+12} m = 10^5\ m = \frac{10^5}{10^3}\ km = 10^2\ km = 100\ km\ (about\ 62\ miles)$$

Example 3:

How many drops of water are in a swimming pool measuring 10 ft × 16 ft × 5 ft if the volume of one drop of water is 0.05 cm^3? (1 foot equal to 30.48 cm)

How many cubic centimeters can fit into this pool?

$$10\ feet = 10 \cdot 30.48\ cm = 304.8\ cm = 3.048 \cdot 10^2\ cm \approx 3.05 \cdot 10^2\ cm$$

$$16\ feet = 16 \cdot 30.48\ cm = 487.68\ cm = 4.8768 \cdot 10^2\ cm \approx 4.88 \cdot 10^2\ cm$$

$$5\ feet = 5 \cdot 30.48\ cm = 152.4\ cm = 1.524 \cdot 10^2\ cm \approx 1.52 \cdot 10^2\ cm$$

$$10 \times 16 \times 5\ cubic\ feet = 3.05 \cdot 10^2 \cdot 4.88 \cdot 10^2 \cdot 1.52 \cdot 10^2\ cm^3 = 22.62368 \cdot 10^{2+2+2} cm^3$$

$$\approx 22.6 \cdot 10^6\ cm^3 = 2.26 \cdot 10^7\ cm^3$$

Now the only thing still unknown is how many drops fit into the volume of the pool.

$$\frac{2.26 \cdot 10^7\ cm^3}{0.05\ cm^3} = \frac{2.26 \cdot 10^7 cm^3}{5 \cdot 10^{-2} cm^3} = \frac{2.26}{5} \cdot \frac{10^7}{10^{-2}} = 0.452 \cdot 10^{7+2} = 4.52 \cdot 10^{-1} \cdot 10^9$$

$$= 4.52 \cdot 10^{9-1} = 4.52 \cdot 10^8$$

Exercises:

1. Express the following numbers as power:

   128; -64; 0.0016; -0.0016; 0.0009; -0.0009

2. Simplify the following expressions:

   a. $3^5 \cdot 3^{-1}$; $5^4 \cdot 5^{-2}$; $3^{-1} : 3^{-2}$; $10^3 : 10^{-3}$; $4^{-2} \cdot 4^{-3}$;

   b. $7^3 \cdot 7^{-2}$; $25^2 \cdot 5^{-2}$; $9^{-1} : 27^{-2}$; $49^4 : 7^3$; $8^{-2} \cdot 16^{-3}$;

$c.\ 3^5 \cdot 3^{-3}$; $16^2 \cdot 4^{-3}$; $27^2 \cdot 3^{-4}$; $4^{-2} : 8^{-2}$; $32^2 \cdot 4^{-3}$;

3. Write in scientific notation:
   Example:
   $2347894532 \approx 2.35 \cdot 10^9$; $0.000675438 = 6.75438 \cdot 10^{-4} \approx 6.75 \cdot 10^{-4}$

   213451213; 0.002315; 123456789; 0.1765;

4. A raindrop has a volume of about 0.05 cm³. How many raindrops are needed to fill a rain barrel that holds 200 liters? 1 liter equal to a $1000\ cm^3$.

5. A drop of water has a volume of 0.05 cm³ and contains about $2 \cdot 10^{21}$ water molecules. About how many water molecules are in a 2-liter bottle of water?

6. A sandbox is 1 m long, 1 m wide, and 20 cm deep. Assume each grain of sand is a cube with an edge length of 0.5 mm. How many grains of sand are needed to fill the sandbox completely? Assume the grains fit together with no gaps. Write your answer in scientific notation.

7. Prove that
   $a.\ 8^5 + 2^{11}$ is divisible by 17, $b.\ 9^7 - 3^{10}$ is divisible by 20
   $c.\ 16^4 + 2^{12}$ is divisible by 17, $d.\ 4^3 - 2^4$ is divisible by 5

Review Problems.

8. A can initially contained twice as much milk as a jar. After 2 liters of milk were poured out of the jar and 3 liters were poured out of the can, the amount of milk remaining in the can was 4.5 times as much as the amount remaining in the jar. How many liters of milk were initially in the jar and in the can?

9. Choose three values of n so that the product is divisible by the given number. Explain which values of n satisfy each condition.:
   a. $3n$ is divisible by 5.
   b. $12n$ is divisible by 4

c. $5n$ is divisible by 15

d. $6n$ is disable by 9.

10. Four people can paint a school auditorium in 10 days. How many days will it take 5 workers to paint the same auditorium if they work at the same rate?

11. There were 24 sunny days in June. What percentage of the days in June were sunny?

12. Evaluate:

$a.\ 1\frac{1}{12}\cdot 1\frac{1}{13}\cdot 1\frac{1}{14}\cdot 1\frac{1}{15}\cdot 1\frac{1}{16}\cdot 1\frac{1}{17};$ $b.\ 4\frac{3}{7}\cdot 8\frac{4}{9}-4\frac{3}{7}\cdot 6\frac{4}{9};$

13. Which expression has the smaller value? Can you determine the answer without performing the calculations? Then evaluate all expressions to check your prediction..

$a.\ 1-\frac{1}{100};$ $b.\ 1-\left(\frac{1}{100}\right)^2;$ $c.\ \left(1-\frac{1}{100}\right)^2$