Prime factorization.

In mathematics, factorization is a decomposition of a number or mathematical expression as a product of numbers or/and expressions.

A number can be represented as the product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 2 \cdot 3 \cdot 6$$

A numerical expression can be written as a product:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Is it possible for any natural number to be expressed as a product of 2 or more numbers other than 1 and itself?

Natural numbers, greater than 1 that has no divisors other than 1 and itself are called **prime numbers**.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization (or integer **factorization)** is a decomposition of a natural number into the product of prime numbers.

Any natural number has single unique prime factorization.

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$, and prime factors of 180 are $2, 2, 3, 3, 5$,

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168; \quad 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

168	2	180	2
84	2	90	2
42	2	45	3
21	3	15	3
7	7	5	5
1		1	

For Halloween, the Jonson family bought 168 mini chocolate bars and 180 gummy worms. What is the largest number of kids among whom the Jonsons can evenly divide both kinds of candy?

To solve this problem, we have to find a number that can serve as a divisor for 168 as well as for 180. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this, they have to find the Greatest Common Divisor (GCD), which is the largest number that can be a divisor for both 168 and 180.

Let's take a look at the set of all prime factors of 168 and 180. For 168 this set contains 2, 2, 2, 3, and 7. Any of these numbers, as well as any of their products, can be a divisor for 168. The same is true for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two representations have common

factors, 2, 2, and 3. This means that both numbers are divisible by any of these common factors and by any of their products. The largest product is the product of all common factors. This largest product has a name: Greatest Common Factor, or GCF. This GCF will also be the Greatest Common Divisor (GCD).

$GCF(168, 180) = 12$



$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$

$168 \div 12 = 14$

$180 \div 12 = 15$



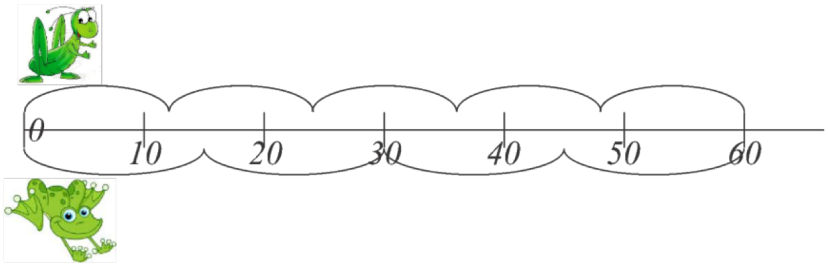
Between 12 kids they can divide both kinds of candy evenly.

A grasshopper jumps a distance of 12 centimeters each leap, while a little frog covers a distance of 15 centimeters per leap. They both start at 0 and hop along the long ruler. What is the closest point on the ruler at which they can meet?

There are specific points on the ruler that both of them can reach after a certain number of leaps. One of these points is, of course, 180 cm.

A grasshopper would make 15 jumps, while a frog would make only 12. Will it be the only place where they can meet or are there other possible meeting points?

Any multiple of 180 will also be divisible by 12 and by 15. Are there any other common multiples of 12 and 15, which are less than 12×15 and are still divisible by 12 and 15?



12	2	15	3
6	2	5	5
3	3	1	
1			

Prime factorization of 12 and 15:

$$12 \cdot 15 = (2 \cdot 2 \cdot 3) \cdot (3 \cdot 5)$$

The number which we are looking for has to be a product of prime factors of 12 and 15.

$2 \cdot 2 \cdot 3 = 12$

$3 \cdot 5 = 15$

$2 \cdot 2 \cdot 3 \cdot 5 = 60$

$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 168$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$

$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2520$

60 is the smallest number, which is divisible by both, 12 and by15, Least Common Multiple (LCM).

$$LCM(12, 15) = 60$$

Homework:

1) Find GCF using prime decomposition:

$a.\ GCF(75, 135);$ $b.\ GCF(180,210);$ $c.\ GCF(125, 462);$

2) Find the LCM using the prime decomposition:

$a.\ LCM(28, 35);$ $b.\ LCM(16, 56);$ $c.\ LCM(21, 100);$