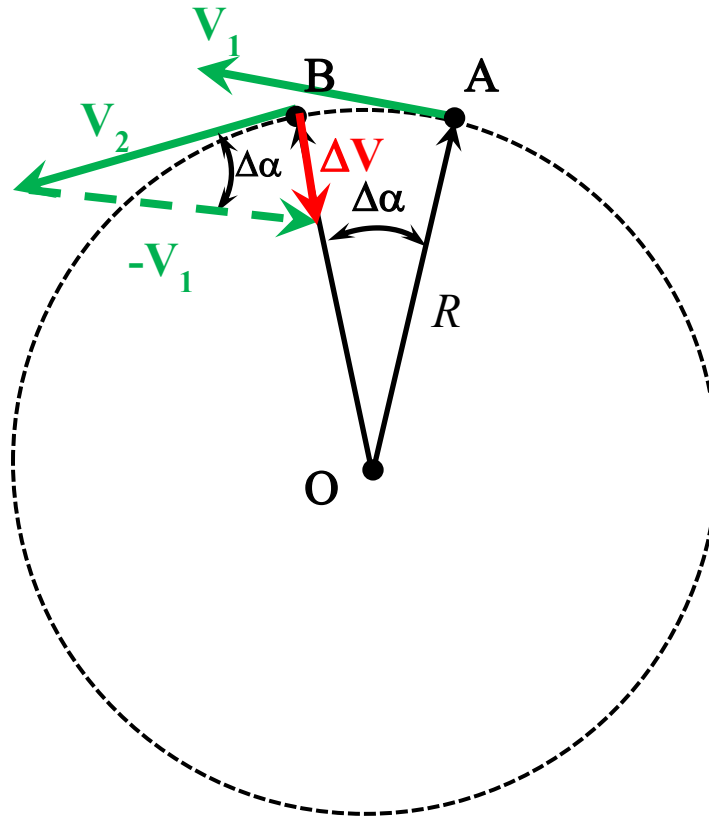


Homework 2

Centripetal acceleration.

We already know that in uniform circular motion, the acceleration vector is inward. How to calculate the magnitude of the centripetal acceleration?



Imagine that an object is moving along a circle from point A to point B (see the picture above). The speed of the object is the same in both points (since it is *uniform* circular motion). The velocity vectors are different. As the object travels from point 1 to point 2, its velocity vector turns. We know that at the circular motion the velocity at any point is directed along the tangent line to this point. The tangent line is perpendicular to the radius drawn from the center to the “tangent” point. Simply speaking, each black arrow in the figure above is perpendicular to the corresponding green arrow. So, the angle $\Delta\alpha$ between the velocities V_1 and V_2 is equal to the angle $\Delta\alpha$, swept by the moving object.

As we remember, acceleration is the change in velocity divided by the time, required for this change. Change in velocity ΔV is shown by the red arrow in the picture above. To find it we have to subtract vector V_1 from the vector V_2 . To do that we will prepare the vector $-V_1$ which has the same length as V_1 , but its direction is opposite. Then we add $-V_1$ to V_2 .

We have to find the length of the red arrow ΔV and divide it by the time Δt which is required for the object to travel from point A to point B.

$$a = \frac{\Delta V}{\Delta t} \quad (1)$$

To calculate ΔV we assume that the arc AB is really really small, so the arc AB is very close to a straight line. We can see that

$$|AB| \approx R \cdot \Delta \alpha \quad (2)$$

It follows from our way to measure the angle. It turns that the formula above is good for any “narrow” isosceles triangle! To find a “short” side we have to multiply one of the “long” sides to the small angle between them. The “narrower” the triangle, the more exact is the formula (2). Let us apply this formula to the triangle formed by V_1 , V_2 and ΔV :

$$\Delta V \approx V \cdot \Delta \alpha \quad (3)$$

Let us plug ΔV from the formula (3) to the formula (1):

$$a = \frac{\Delta V}{\Delta t} \approx \frac{V \cdot \Delta \alpha}{\Delta t} \quad (4)$$

But $\Delta \alpha / \Delta t$ is the angular velocity ω . So we can write:

$$a = V \cdot \omega \quad (5)$$

If we remember that $\omega = V/R$, the magnitude of the acceleration can be written as

$$a = \frac{V^2}{R} \quad (6),$$

where R is the radius of the circle. So we can see that the faster we move and the sharper we turn the more acceleration we experience and the more centripetal force we need to complete the turn.

Now, as long as we know that an object moves along a circle with a radius R with a constant speed V , we can immediately write its acceleration using the formula (6)!

Problems:

1. A 1200 kg car travels around a **flat circular curve** of radius 50 m. The coefficient of static friction between the tires and the dry road is $\mu_s = 0.65$. Assume $g = 9.8 \text{ m/s}^2$.

a) Draw a free-body diagram of the car. Identify the force that provides the centripetal acceleration.

b) Calculate the maximum static friction force available between the tires and the road.

c) Determine the **maximum speed** at which the car can travel around the curve without skidding.

d) A student claims that a 2400 kg truck with the same tires and the same coefficient of friction must take the curve more slowly because it has twice the mass. Is this correct? Support your answer mathematically.

e) It begins to rain, and the coefficient of static friction decreases to 0.30. Find the new maximum safe speed.

f) Without doing a complete calculation, determine by what factor the maximum speed changes when the coefficient of friction changes from 0.65 to 0.30.

Challenge: The road is redesigned so that the radius of the curve is doubled to 100 m, while $\mu_s = 0.65$. By what factor does the maximum possible speed increase?

2. A 0.40 kg ball is attached to a 1.20 m string and swung in a horizontal circle. The string can withstand a maximum tension of 30 N.

Assume initially that the string remains horizontal.

a) If the ball moves at 5.0 m/s, calculate its centripetal acceleration.

b) Calculate the force required to produce this centripetal acceleration. Will the string break?

c) Determine the **maximum speed** at which the ball can move without breaking the string.

d) Suppose the string is replaced by one of the same strength but only 0.60 m long. Determine the new maximum speed.

e) Compare your answers from (c) and (d). Explain why decreasing the radius decreases the maximum allowable speed even though the shorter path might initially seem to make circular motion "easier."