

Homework 1

Circular motion. Centripetal acceleration. Angular velocity.

We are discussing circular motion. We already know that any motion along a curved line is accelerated motion. I would say that the simplest example of a curved line is a circle. Studying of the circular motion is even more important and general than it seems, because any smooth curved line can be composed from “pieces” of circles with different diameters. So the motion of an object along such curve at any moment of time can be represented as circular motion. A convenient way to describe the position of an object moving along a circular path is to measure the angle (let us denote it α) swiped by the radius connecting the object and the center of the circular path.

A good way to measure angle.

There is a convenient way to measure the angle. We can divide the length of the circular arc swiped by the radius of the circle to the radius. The obtained quotient does not depend on the radius of the circle. The unit of the angle measured this way is called *radian*. For example the angle of 360° , corresponding to one complete turn is: $\alpha = 2\pi R/R = 2\pi$ rad. So one radian is approximately equal to 57.3° .

Since circular motion is accelerated motion it does not persist without a force.

1. An overhead view of a person swinging a rock on a rope. A force from the string is required to make the rock's velocity vector keep changing direction.
2. If the string breaks, the rock will follow Newton's first law and go straight instead of continuing around the circle.

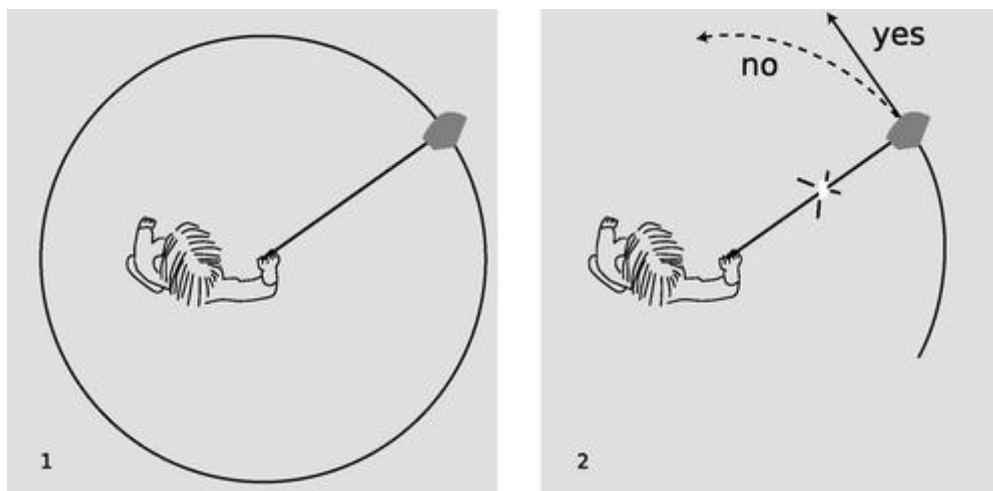


Figure 1 (www.lightandmatter.com)

The force is directed inward and called *centripetal* force.

Figure 2 showed the string pulling in straight along a radius of the circle, but many people believe that when they are doing this they must be “leading” the rock a little to keep it moving along. That is, they believe that the force required to produce uniform circular motion is not directly inward but at a slight angle to the radius of the circle. This intuition is incorrect, which you can easily verify for yourself now if you have some string handy. It is only while you are getting the object going that your force needs to be at an angle to the radius. During this initial period of speeding up, the motion is not uniform. Once you settle down into uniform circular motion, you only apply an inward force.

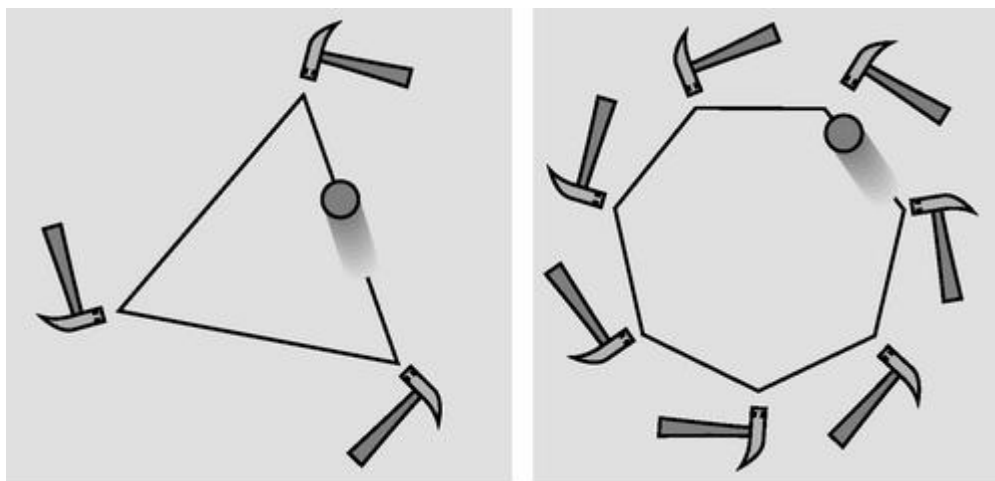
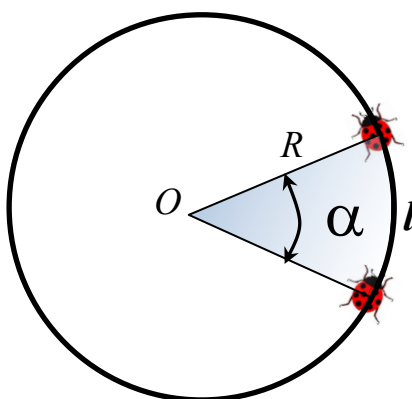


Fig. 3 (www.lightandmatter.com)

A series of three hammer taps makes the rolling ball trace a triangle, seven hammers a heptagon. If the number of hammers was large enough, the ball would essentially be experiencing a steady inward force, and it would go in a circle. In no case is any forward force necessary.

Angular velocity

According to the definition, angular speed is the angle “swept” by a revolving object per unit time. (Just to be specific: we are using the word “rotation” if the axis passes through the rotating object. In case the axis is outside the object we will use the word “revolution”). Let us assume that the ladybug performs uniform revolution around axis O and sweeps angle α during the time t .



We can write the angular speed ω as:

$$\omega = \frac{\alpha}{t} \quad (1)$$

According to our definition of angle magnitude, the angle α equals to the length of the circular arc l divided by the radius of the circle R .

$$\alpha = \frac{l}{R} \quad (2)$$

If we replace the angle α in the expression 1 for l/R , we will obtain:

$$\omega = \frac{l}{R \cdot t} \quad (3)$$

But the length of the arc l is the distance, passed by the ladybug during the time t . So l/t is the linear speed v of the ladybug. Taking this into account we can write (3) as:

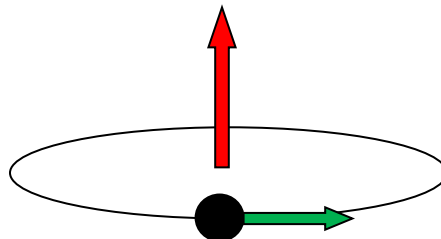
$$\omega = \frac{v}{R} \quad (4)$$

This is the expression, connection linear and angular speeds of the revolving object. We can see and the smaller radius at the same linear speed, the higher angular speed.

The time T which is required to complete one full turn is called the *period* of revolution. .Since in this case the angle passed is 2π ,

$$T = \frac{2\pi}{\omega} \quad (5).$$

We have been discussing angular speed which does not have direction. What direction should we choose for the angular velocity? There are two ways for an object to move along a circular path with respect to us: clockwise and counterclockwise (these “directions” are swapped if we will be looking from the other side of the plane of the orbit). The vector of angular velocity is parallel to the axis of revolution (which means that it is perpendicular to the orbit plane). If we are “looking” from the end of the angular velocity vector, we will see that the object is revolving counterclockwise.



$$\vec{v}$$

There is a convenient way to remember the relation between the directions of linear and angular velocities. It is called “right-hand rule”:



Even if the angular velocity does not change, the motion is accelerated and acceleration is directed toward the center of the circular trajectory (centripetal acceleration)

Problem:s:

Problem 1 — Two points on a rotating disk

A circular platform of radius 1.20 m rotates uniformly at **15 revolutions per minute (rpm)**. Two students stand on the platform. Anna is 0.40 m from the axis, while Ben stands at the outer edge.

- Find the angular velocity of the platform in rad/s.
- Find the linear velocities of Anna and Ben.
- Find the ratio v_B/v_A .

Problem 2 — Bicycle wheels of different sizes

A bicycle moves along a straight road without slipping at 7.0 m/s. The radius of the rear wheel is 0.35 m, while the radius of the front wheel is 0.28 m. Find the angular velocity of each wheel.

Problem 3 — Two connected rotating wheels

Two wheels are connected by a belt that does not slip. Wheel A has radius $R_A = 10$ cm, and wheel B has radius $R_B = 25$ cm. Wheel A rotates at **180 rpm**.

- Find the angular velocity of wheel A in rad/s.
- Find the linear velocity of the belt.
- Determine the angular velocity of wheel B in rad/s and in rpm.