

MATH 5e

Topics: More on powers. Arithmetic operations

- Powers notation

General notation (n is a whole number):

$a^n = a \times a \times a \times \dots \times a$ (n times). a is called the base, and n – the exponent

Special cases:

$a^0 = 1$ read: a -to-the-zero

$a^1 = a$ is just itself ‘ a ’

$a^2 = a \times a$ read: a -squared

$a^3 = a \times a \times a$ read: a -cubed

- Multiplication and division of powers with the same base

$$a^n a^m = a^{n+m}$$

Because:

$$a^n a^m = \underbrace{(a \times a \times a \dots)}_{n\text{-times}} \times \underbrace{(a \times a \times a \dots)}_{m\text{-times}} = \underbrace{a \times a \times a \times a \dots}_{n+m \text{ times}}$$

$$a^n : a^m = \frac{a^n}{a^m} = a^{n-m} \quad \text{if } n > m \quad \text{or} \quad \frac{a^n}{a^m} = \frac{1}{a^{m-n}} \quad \text{if } m > n$$

$$a^n = \frac{1}{a^{-n}}$$

$$a^{-n} = \frac{1}{a^n}$$

Then we define $\frac{a^n}{a^n} = a^{n-n} = a^0 = 1$

- Power of a product

$$(ab)^n = a^n \times b^n$$

because

$$(ab)^n = ab \times ab \times ab \times \dots \times ab \text{ (} n \text{ times)} = (a \times a \times a \times \dots \times a) \times (b \times b \times b \times \dots \times b) \text{ (} n \text{ times)}$$

- Power raised on a power

$$(a^m)^n = a^{m \times n}$$

example

$$(a^2)^3 = a^2 \times a^2 \times a^2 \text{ (3 times)} = a \times a \times a \times a \times a \times a \text{ (6 times)} = a^6$$