Homework 23.

Huygens-Fresnel principle

We will discuss another phenomenon related to the wave propagation – *diffraction*. One can describe diffraction in a simplest way as the ability of a propagating wave to bend around the edges of an obstruction. Diffraction can be explained in the frame of Huygens-Fresnel principle of wave propagation. It can be formulated as follows:

"Each point of the leading surface of a wave disturbance- the wave front- may be regarded as a secondary source of spherical waves (or *wavelets*) , which themselves progress with the speed of light in the medium and whose envelope at a later time constitutes a new wavefront" – from Frank. L. Pedrotti, S.J., Leno S. Pedrotti "Introduction to Optics" Prentice Hall, NJ, 1987.

Illustration of this principle is given in Figure 1, where propagation of a plane and spherical waves is shown.

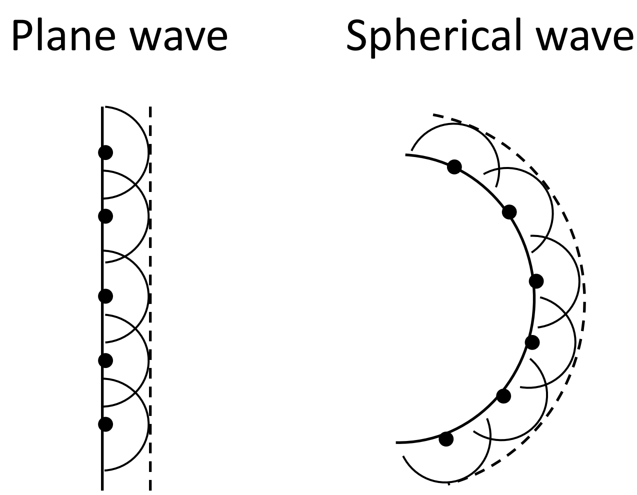


Figure 1. Propagation of a plane and spherical waves.

The words "plane" and "spherical" are related to the shape of the wave front – the locus of the points oscillating in the same phase. Each semicircle in Figure 1 represents a wavelet. The radius of a wavelet in a time t after the wavelet onset is the product of the speed of light $\boldsymbol{c}$ in the media and the time $\boldsymbol{t}$ (Figure 2).

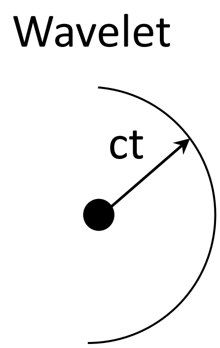


Figure 2. Wavelet.

As the wavefront reaches an obstruction, for example a screen with a hole, each point of the media inside the hole emits a wavelet (Figurev 3).

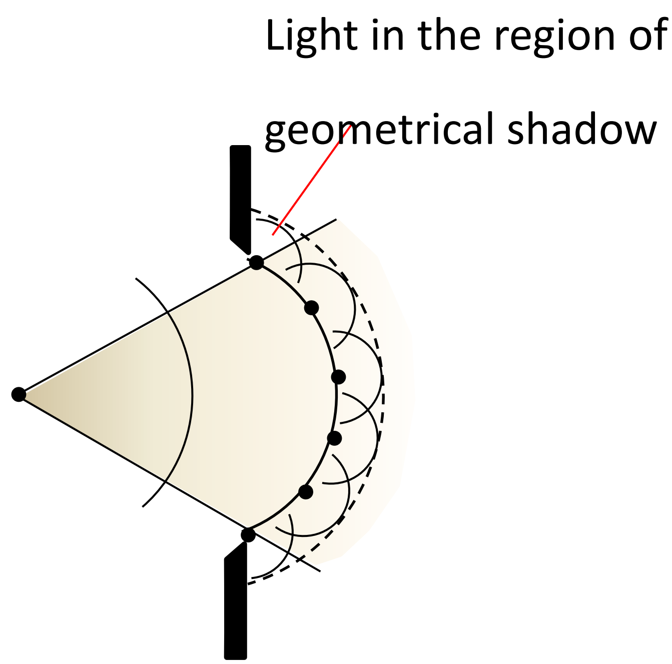


Figure 3. Huygens principle applied to an obstructed spherical wave.

As we see from Figure 3, due to diffraction, the light can penetrate to the region of geometrical shadow, i.e. beyond the cone restricted by two straight lines.

Dutch physicist Christiaan Huygens, who put forward the principle could not accept the fact that the light can be “bended” hear the edge of an obstruction. Neither could he explain why we have to consider only half of the wavelet. The principle was later amended by French engineer and physicist Augustin-Jean Fresnel and others.

Christiaan Huygens

1629-1695

Augustin-Jean Fresnel

1788-1827

Huygens-Fresnel principle can be used to obtain basic laws of light propagation. Below (Figure 5) is a Huygens construction which illustrates one of the basic optical laws: the law of reflection: the reflection angle Θ_r is equal to the incidence angle Θ_i.

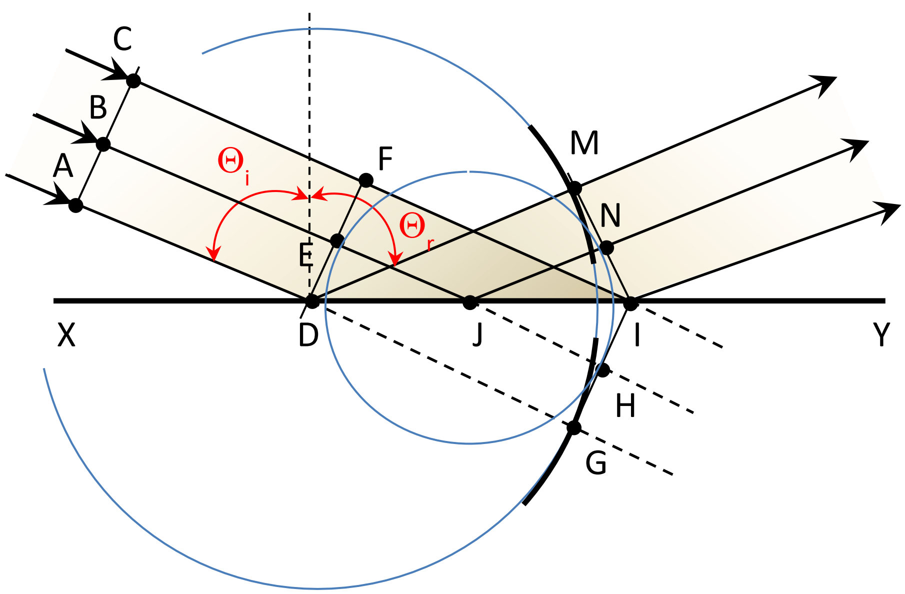


Figure 5. Huygens construction to prove the law of reflection.

To explain Figure 5 we will follow the way suggested by Frank and Leno Pedrotti in their book "Introduction to Optics". A plane wavefront AC encounters a reflecting interface XY at an angle of incidence Θ_i. As we can see from Figure 5, the points of wavefront AC will reach the interface at different time. If the interface XY were not present, at the time point C would reach position I, the wavefront AC would have occupied position GI. The intrusion of the reflecting surface means that as point F progresses to point I, point E progresses to point J and, then, the distance equal to JH but after reflection, so $|JH| = |JN|$. Similarly, $|DG| = |DM|$. So, the wavefront MN of the reflected wave is tangent to two blue circles: the big one with the center in D and radius DM and the smaller one, with the center in J and radius JN. Now :

1) $\angle ADX = \angle IDG$ (vertical angles),

2) two right triangles DMI and DIG are equal (since they have same hypotenuse DI and DM=DG, then MI=IG according to the Pythagorean theorem, so this is SSS congruency);

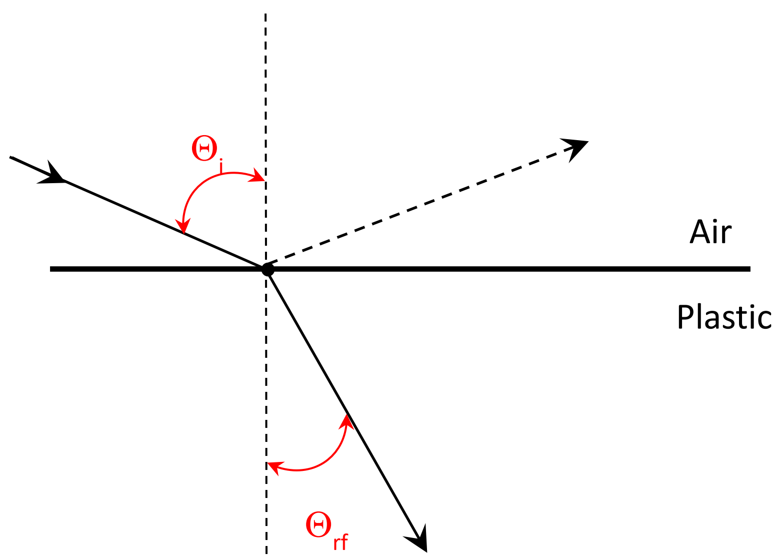
3) from (1) and (2) ⇒ $\angle ADX = \angle IDM$

4) from (3) ⇒ $\Theta_i = \Theta_r$

Problem:

As we know from numerous observations, the direction of light propagation generally changes as the light travels from one media to another. This phenomenon is called ***refraction***. Try to obtain Snell's refraction law using Huygens' principle.

Given: plain wavefront of light travels in the air and encounters the plain surface of a thick clear plastic slab. The speed of light in the plastic is **n** times lower than this in the air.



Prove that $\sin \Theta_i = n \cdot sin\Theta_{rf}$. Θ_{rf} is called ***angle of refraction***. Make a picture of the wavefronts.