Homework 1

Circular motion. Centripetal acceleration.

We are discussing circular motion. We already know that any motion along a curved line is accelerated motion. I would say that the simplest example of a curved line is a circle. Studying of the circular motion is even more important and general than it seems, because any smooth curved line can be composed from “pieces” of circles with different diameters. So the motion of an object along such curve at any moment of time can be represented as circular motion.

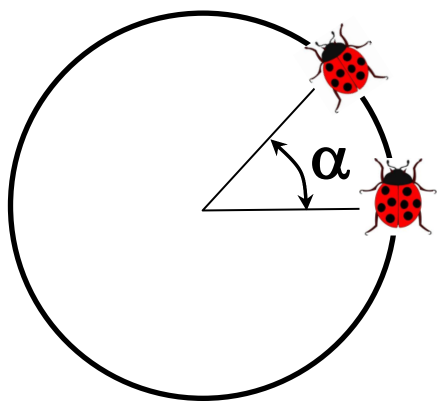


Figure 1.

A convenient way to describe the position of an object moving along a circular path is to measure the angle (let us denote it α) swiped by the radius connecting the object and the center of the circular path.

A good way to measure angle.

There is a convenient way to measure the angle. We can divide the length of the circular arc swiped by the radius of the circle to the radius. The obtained quotient does not depend on the radius of the circle. The unit of the angle measured this way is called *radian*. For example the angle of 360^o, corresponding to one complete turn is: $\boldsymbol{\alpha = 2\pi R/R = 2\pi}$ rad. So one radian is approximately equal to 57.3^o.

Since circular motion is accelerated motion it does not persist without a force.

1. An overhead view of a person swinging a rock on a rope. A force from the string is required to make the rock's velocity vector keep changing direction.

2. If the string breaks, the rock will follow Newton's first law and go straight instead of continuin around the circle.

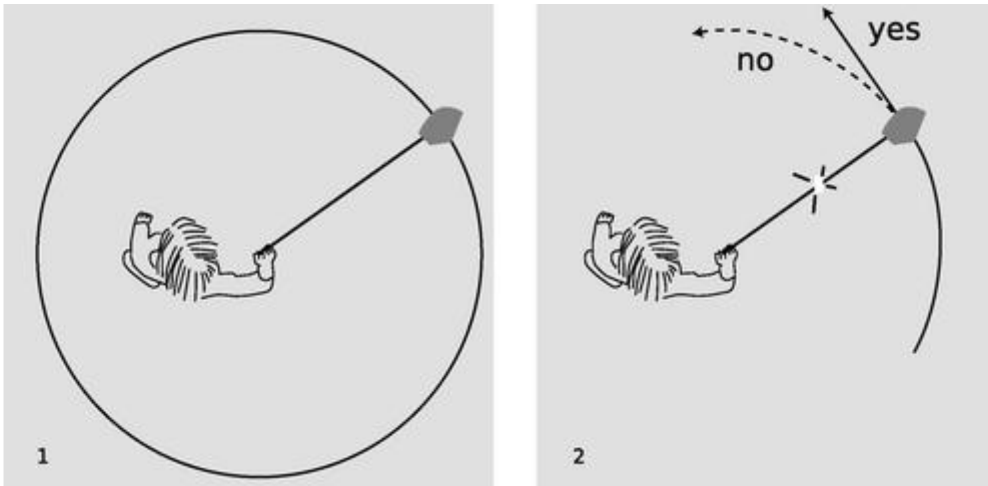


Figure 2 (www.lightandmatter.com)

The force is directed inward and called *centripetal* force.

Figure 2 showed the string pulling in straight along a radius of the circle, but many people believe that when they are doing this they must be "leading" the rock a little to keep it moving along. That is, they believe that the force required to produce uniform circular motion is not directly inward but at a slight angle to the radius of the circle. This intuition is incorrect, which you can easily verify for yourself now if you have some string handy. It is only while you are getting the object going that your force needs to be at an angle to the radius. During this initial period of speeding up, the motion is not uniform. Once you settle down into uniform circular motion, you only apply an inward force.

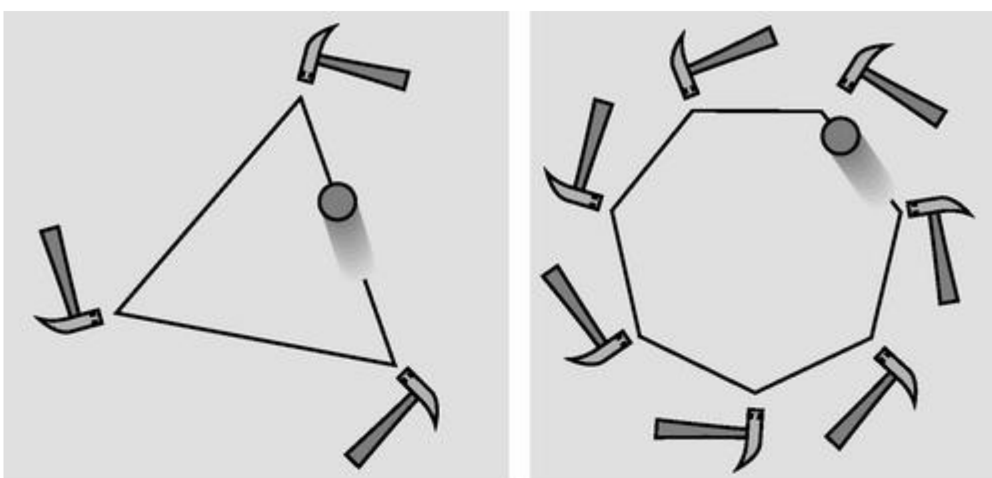
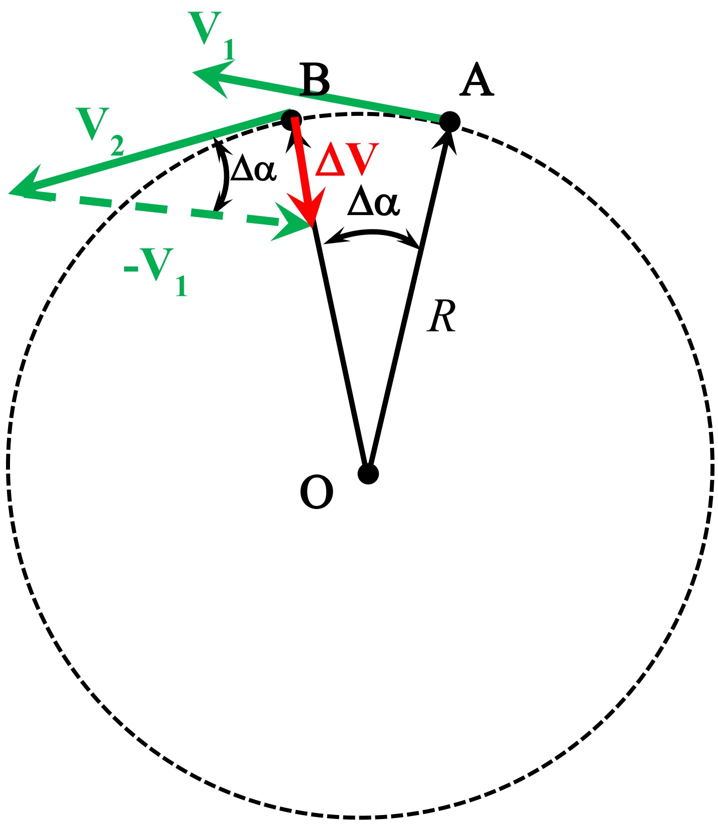


Fig. 3 (www.lightandmatter.com)

A series of three hammer taps makes the rolling ball trace a triangle, seven hammers a heptagon. If the number of hammers was large enough, the ball would essentially be experiencing a steady inward force, and it would go in a circle. In no case is any forward force necessary.

Centripetal acceleration.

We already know that in uniform circular motion, the acceleration vector is inward. How to calculate the magnitude of the centripetal acceleration?



Imagine that an object is moving along a circle from point A to point B (see the picture above). The speed of the object is the same in both points (since it is *uniform* circular motion). The velocity vectors are different. As the object travels from point 1 to point 2, its velocity vector turns. We know that at the circular motion the velocity at any point is directed along the tangent line to this point. The tangent line is perpendicular to the radius drawn from the center to the "tangent" point. Simply speaking, each black arrow in the figure above is perpendicular to the corresponding green arrow. So, the angle $\Delta\alpha$ between the velocities V_1 and V_2 is equal to the angle $\Delta\alpha$, swept by the moving object.

As we remember, acceleration is the change in velocity divided by the time, required for this change. Change in velocity ΔV is shown by the red arrow in the picture above. To find it we have to subtract vector ***V1*** from the vector ***V2***. To do that we will prepare the vector ***–V1*** which has the same length as ***V1,*** but its direction is opposite. Then we add ***–V1*** to ***V2***.

We have to find the length of the red arrow ΔV and divide it by the time Δt which is required for the object to travel from point A to point B.

$$a = \frac{\Delta V}{\Delta t} \quad (1)$$

To calculate ΔV we assume that the arc AB is really really small, so the arc AB is very close to a straight line. We can see that

$$|AB| \approx R \cdot \Delta\alpha \quad (2)$$

It follows from our way to measure the angle. It turns that the formula above is good for any “narrow” isosceles triangle! To find a “short” side we have to multiply one of the “long” sides to the small angle between them. The “narrower” the triangle, the more exact is the formula (2). Let us apply this formula to the triangle formed by V_1, V_2 and ΔV:

$$\Delta V \approx V \cdot \Delta\alpha \qquad (3)$$

Let us plug ΔV from the formula (3) to the formula (1):

$$a = \frac{\Delta V}{\Delta t} \approx \frac{V \cdot \Delta\alpha}{\Delta t} \qquad (4)$$

But $\Delta\alpha/\Delta t$ is the angular velocity $\boldsymbol{\omega}$. So we can write:

$$a = V \cdot \omega \quad (5)$$

If we remember that $\boldsymbol{\omega = V/R}$, the magnitude of the acceleration can be written as

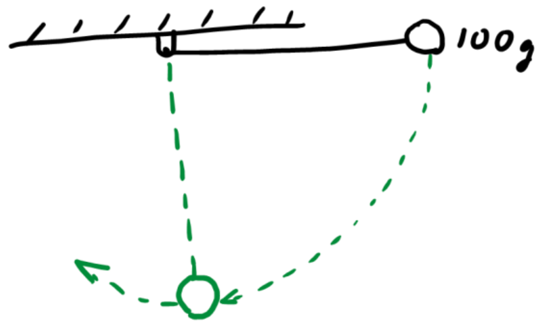
$$a = \frac{V^2}{R} \quad (6),$$

where R is the radius of the circle. So we can see that the faster we move and the sharper we turn the more acceleration we experience and the more centripetal force we need to complete the turn.
Now, as long as we know that an object moves along a circle with a radius R with a constant speed V, we can immediately write its acceleration using the formula (6)!

Problems:

1. You are staying at the equator. Calculate your linear speed with respect to the center of Earth

2. You release a 100g bob of a 1m long pendulum while the pendulum is in horizontal position (see Figure below). The maximum tension of the thread is 20N. Will the thread break?



3. A car is moving through a semicircle bridge with a radius of 20m at a speed of 20m/s. (This is a very fast car!). What is the weight of the car at the upper point of the bridge? Try to explain what will be happening to the car as it moves across the bridge.