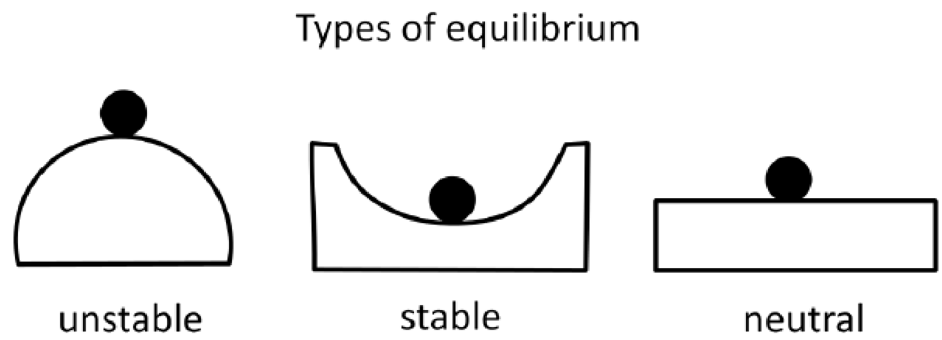
Homework 11

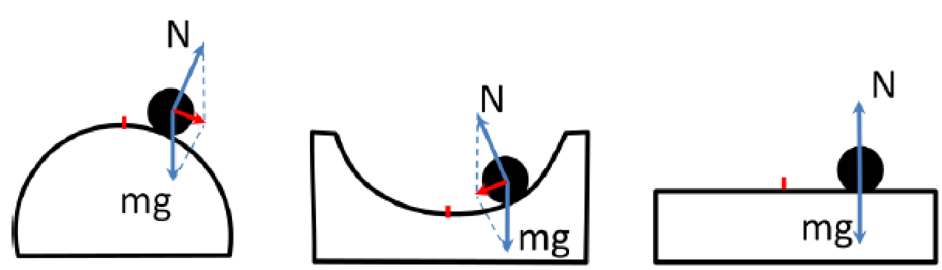
Oscillations

Generally, oscillation can be described as periodic (or, even more generally just repetitive) variations of a certain parameter. Heartbeat, Earth rotation, pendulum, 11-year Sun activity cycle, light – these are examples of oscillatory phenomena.

In which systems we can observe oscillatory behavior? To understand this let us consider three systems in equilibrium (see Figure below):



If we slightly displace the ball which is on top of the hill (left picture) it will roll down because the total net force will be directed from the equilibrium position. For the ball displaced from the center of the bottom of a spherical bowl the force is directed back to the equilibrium point. We will call such force as *restoring force*. In case of neutral equilibrium, no force appear and the ball just stay in new position (see Figure below).



Systems in stable equilibrium can demonstrate oscillatory behavior.

We will be considering only periodic mechanical oscillations. May be a simplest example of a mechanical oscillator is a mass attached to the coil spring. If we pull the mass and then let it go, the coordinate of the mass will be changing in a periodical way around the equilibrium position. The number of oscillations per unit time is called ***frequency (ν)***. The unit of frequency is 1Hz (Hertz) – one oscillation per second. It is named after German physicist Heinrich Rudolf Hertz.

Heinrich Rudolf Hertz (1857-1894).

In some cases it is convenient to use ***angular frequency (ω)***– the number of oscillations (or rotations) per 2π seconds.

$$\omega = 2\pi \cdot v \quad (1)$$

The time which is necessary to complete one full oscillation is called ***period*** (***T***). It is easy to see that T=1/ν. Maximum deviation of the coordinate from the equilibrium point during the oscillatory motion is called ***amplitude*** (***A***).

The oscillations are called "harmonic" when the restoring force applied to the oscillating object is proportional to the displacement of the object to the equilibrium point. The simplest example of such a system is the mass m attached to the coil spring non a frictionless surface. Restoring force in this case is :F=-kx (Hook's law), where k is the elastic coefficient of the coil spring, x is the displacement of the mass. So, using the second Newton's law we can write:

$$ma = -kx \Rightarrow a + \frac{k}{m}x = 0. \quad (2)$$

This is the equation of harmonic oscillation. We do not know yet how to solve such an equation. The difficulty is that now the acceleration *a* depends on the coordinate. Usually we worked with a constant acceleration and used this acceleration to find the coordinate at any moment. Now the acceleration changes from point to point. The acceleration ***a*** and the coordinate **x** are not independent. The acceleration is the rate of change of the velocity while the velocity is the rate of change of the coordinate. So the equation 2 can be solved, but we need calculus to do that. At this point you have just to believe that this equation leads to periodic motion with the period:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (3)$$

So whenever you see the equation of motion in the form of:

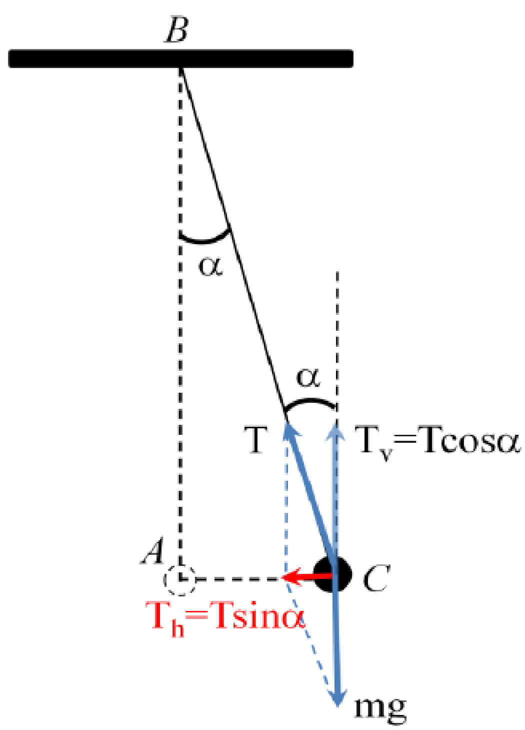
$$a + K \cdot x = 0 \quad (4),$$

where a is the acceleration, x – coordinate and K is coefficient which does not depend on x and time t, you can be sure that it describes periodic oscillations and the period is:

$$T = \frac{2\pi}{\sqrt{K}} \quad (5)$$

Example:

Let us consider the pendulum.



The length of the pendulum is l, T is the tension force of the thread, point A is the equilibrium position. The second Newton's law for the vertical axis (positive direction is up) is:

$$ma_v = T\cos\alpha - mg \quad (6)$$

here a_v is the vertical component of the acceleration, m is the mass of the bob. For the horizontal axis (positive direction is left to right) the second Newton's law is:

$$ma_h = -T\sin\alpha \quad (7)$$

Segment $|AC| = l \cdot sin\alpha$ is horizontal displacement x. So, we can write (7) as:

$$ma_h = -\frac{T}{l}x \quad (8)$$

Here is very important point. Let us consider only very small deflections of the pendulum from the equilibrium position, so angle α is very very small and the bob moves horizontally. So we can take the vertical component of the acceleration equal to zero:

$$a_h = a;\ a_v = 0 \quad (9)$$

Moreover, for small angles we can further simplify the equations, because for small angles**$\sin\alpha \approx \alpha$,** and**$\cos\alpha \approx 1$.** You can check these expressions using calculator. I would like you to memorize them, because they are very useful and widely applied in physics. So for the vertical axis we have:

$$T - mg = 0 \quad (10)$$

For the horizontal axis the simplified expression of the second Newton's law is:

$$ma = -\frac{T}{l}x \quad (11)$$

We can find T from equation 10 and plug it to equation 11. Then we will divide both parts of the equation 11 to m. The result is

$$a + \frac{g}{l}x = 0$$

The equation is valid for small deviation of the pendulum's bob from the equilibrium point.

The oscillation period is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Problem 1.

An empty cylinder of radius **R** spinning as an angular velocity **ω** is placed on the floor next to the wall (Figure 1). The friction coefficient between the cylinder, the wall and the floor is **μ**. How many turns will the cylinder complete before it stops?

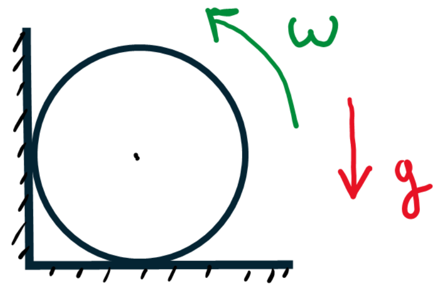


Figure 1.

Problem 2.

A bead of mass **m** is fixed in the center of a weightless string of length **L**.The string is under tension force **T**. Find the period of small oscillations of the bead in the horizontal direction (Figure 2).

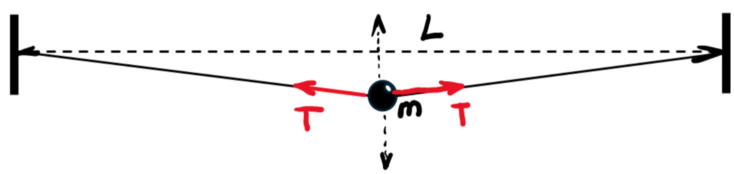


Figure 2.

Problem 3.

A wooden bar of mass m is placed on 2 rapidly rotating cylinders, separated by a distance L (Figure 3). The friction coefficient between the bar and the cylinders is m. Find the period of small oscillations of the wooden bar.