

Acceleration 3

Motion at constant acceleration

Recall from the previous class that accelerated motion occurs whenever velocity changes. If velocity changes at a constant rate, this means that acceleration is constant and velocity ($\vec{V}$) dependence on time (t) is very simple:

$$\vec{V}_{final} = \vec{V}_{initial} + \vec{a}t$$

The average velocity of accelerated movement is:

$$V_{average} = \frac{V_{initial} + V_{final}}{2} = \frac{V_{initial} + V_{initial} + at}{2} = \frac{2 \cdot V_{initial} + at}{2} = V_{initial} + \frac{at}{2}$$

The next thing we would like to know is how far do we travel when accelerating constantly. We considered an application of this question: if we want to explore a vertical cave but do not see the bottom and are not sure that our rope will be long enough. To measure the depth of the cave before going in we can throw a rock there and measure the time it takes to reach the bottom (which we would hear). It would be smart to “throw” the pebble with zero initial velocity, simply let it go from our hand. Then we need to find out how to relate the distance traveled by the pebble to time of travel.

Here $\vec{V}_0$ is initial velocity and $\vec{a}$ is acceleration.

For motion at constant acceleration a , with no initial speed, the displacement after time t is:

$$\bar{D} = \overline{V_{average}} * t = \frac{\bar{a} * t}{2} * t = \frac{\bar{a} * t^2}{2}$$

Example:

If it took our pebble 3 seconds to fall, we calculate the depth of the cave to be

$$d = \frac{gt^2}{2} = \frac{10 \cdot 3^2}{2} \text{ m} = 45 \text{ m}$$

So, a standard 50 m rope will be enough.

Now assume that an object moves **with acceleration $\vec{a}$ while having a non-zero initial velocity $\vec{V}_0$**). Then displacement is:

$$\vec{D} = \overrightarrow{V_{average}} \cdot t = \left(\overrightarrow{V_{initial}} + \frac{\vec{a}t}{2} \right) \cdot t = \vec{V}_{initial} \cdot t + \frac{\vec{a} \cdot t \cdot t}{2} = \vec{V}_{initial} \cdot t + \frac{\vec{a} \cdot t^2}{2}$$

For a negative acceleration (if the car stops, i.e. the acceleration is directed opposite to the velocity) we have:

$$D = V_{initial} \cdot t - \frac{a \cdot t^2}{2}.$$

Homework:

1) A coin has been falling for 3 sec. An initial velocity of the coin was 0. Find the displacement of the coin during the third second.

2) You have a bet with your friend that you could throw a ball higher than the roof of your school. Your school is 6 m high. You threw the ball vertically up with initial speed 15 m/s.

(a) In what time will it reach the highest point? (hint: at highest point the ball has to stop)

(b) What height will the ball reach? Did you win the bet?

3) When driving a car at night with low beam headlights on, the driver can see the road up to 30 meters ahead. The driver suddenly sees a deer crossing the road ahead within the headlight reach. He immediately slams the brakes, and the car starts braking with acceleration -5 m/s². At what maximal initial speed can the car still avoid hitting the deer? Convert your answer to miles per hour.