

# MOMENTUM AND MOMENTUM CONSERVATION

## THEORY RECAP

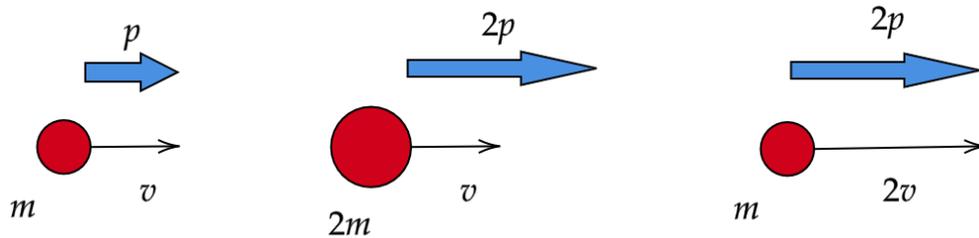
Imagine that a car and a big truck are moving with the same speed. Which one of them is harder to stop? One way to answer this question is from the point of view of Newton's second law. The truck has larger mass, so to supply it with the same acceleration as the car, one needs to apply larger force.

There is another reasoning leading to the same conclusion. It is related to Newton's second law but formulated differently. We introduce a new quantity called momentum which is equal to product of mass and velocity:

$$\vec{p} = m\vec{v}$$

We see that for larger mass and for larger speed momentum gets larger. A truck has larger momentum than a car, so it is harder to stop the truck than the car.

As we see from the definition of momentum, it is a vector. It has magnitude equal to product of mass and speed and its direction is the same as direction of velocity. This is illustrated on a figure below.



Units of momentum are the product of units of mass and speed:  $\text{kg} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$ . Unlike force, there is no special name for this unit.

Since momentum is a product of mass and velocity, if we change either mass or velocity of an object, its' momentum will change. Moving objects with changing mass are somewhat exotic, so we will not consider it and will focus on change in velocity. Change in velocity requires acceleration. By Newton's second law acceleration requires a force. So, change in momentum requires some force. Note that we do not say anything new here, it is Newton's first law expressed in different terms. If no net force is applied to a body, its momentum stays the same.

How momentum changes when there is a force will be discussed next time. Today we will look at momentum of a system of bodies instead. Say we have two blocks, one with mass  $m_1$  and velocity  $\vec{v}_1$  and the other with mass  $m_2$  and velocity  $\vec{v}_2$ . We can find their total momentum by adding their individual momenta:

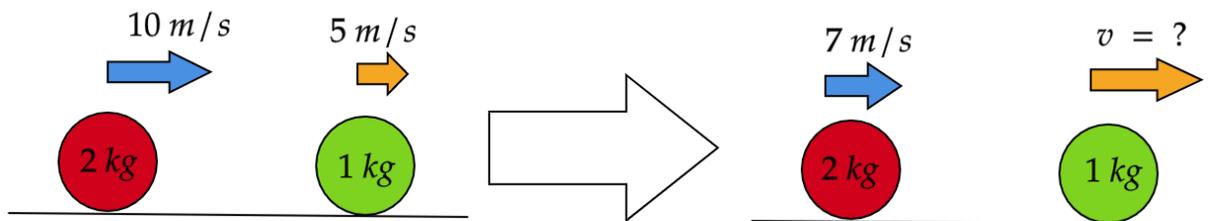
$$\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Remember that for one body if there is no net force, its momentum stays constant. There is a generalization of this to our system: if there is no net external force, the total momentum of a system stays constant.

First, let us understand what external force means. We have two bodies in our system. They interact with each other and with some external objects (the Earth, floor, etc.). Forces of interaction between the objects in the system are called internal. They *always* come in pairs of equal and opposite forces: if  $m_1$  acts on  $m_2$  with force  $\vec{F}$  then by Newton's third law  $m_2$  acts on  $m_1$  with force  $-\vec{F}$ . Therefore, when we consider the whole system and add up all the forces, these internal forces *always* cancel out. On the other hand, forces between the objects in the system and external objects, called external forces, do not cancel out. So only external forces could change properties of the whole system, such as total momentum. If there is no net external force, momentum does not change.

The statement that some quantity does not change (stays constant in time, is conserved) is called a conservation law. We have formulated **momentum conservation law**: total momentum is conserved if there is no net external force. It is a very-very important law. There are many cases when using it is much simpler than using Newton's laws directly. Moreover, though we will not encounter this in our class, momentum conservation law still stays applicable even when Newton's aren't applicable anymore - like in relativity theory and in quantum theory.

Let us apply momentum conservation law to study a collision of two balls on a horizontal table without friction. The balls have masses  $m_1 = 2 \text{ kg}$  and  $m_2 = 1 \text{ kg}$ . Before the collision they respectively had velocities  $v_1 = 10 \text{ m/s}$  and  $v_2 = 5 \text{ m/s}$  directed to the right. After the collision the  $m_1$  ball has velocity  $v'_1 = 7 \text{ m/s}$  directed to the right. Let us find the velocity of the  $m_2$  ball after the collision.



The main idea is that since there is no friction, there is no net external force (in the vertical direction gravity is balanced by the normal force). Therefore total momentum is conserved. Let us write total momentum before the collision choosing positive direction to the right:

$$p_{tot} = p_1 + p_2 = m_1 v_1 + m_2 v_2 = 25 \text{ kg} \cdot \frac{\text{m}}{\text{s}}$$

After the collision the total momentum is expressed via new velocities:

$$p_{tot} = p'_1 + p'_2 = m_1 v'_1 + m_2 v = 14 + v$$

By setting one  $p_{tot}$  equal to the other we get

$$25 = 14 + v \implies v = 11 \text{ m/s}$$

## HOMEWORK

- 1) Two cars with mass 1000kg and 2000kg go toward each other. The speed of the first car is 50km/h, the speed of the second is 40 km/h. Find the total net momentum of the two cars. Make a picture.
- 2) A block of mass  $M = 100$  g moves with the speed of  $v = 10$  m/s on a frictionless flat surface. A bullet of mass  $m = 8$  g moves with the speed  $u = 700$  m/s in the opposite direction, hits the block and gets stuck in it. What will be the velocity of the block after this collision (include direction in your response)?
- 3) Compare the work done by a car's engine to accelerate the car from 0 km/h to 27 km/h with the work which is necessary to accelerate the car from 27 km/h to 54 km/h.