Homework 5

How to calculate displacement of a uniformly accelerated object

Last class we have also learned how to calculate the displacement of a uniformly accelerated object. Uniform acceleration (constant acceleration) means that the acceleration does not change as the object is moving.

Example

A car spent time *t* moving with positive acceleration *a* from point A to point B along a straight line. A velocity of the car at point A was $\vec{V}_{initial}$. In time *t* after the car started from the point A its velocity is

$$\vec{V}_{final} = \vec{V}_{initial} + \vec{a}t \qquad (1)$$

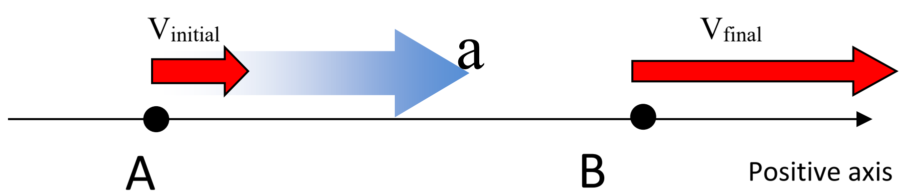


Figure 1.

After we have chosen our "positive" axis, we can drop arrows on top of the vectors, but at the same time we have to choose the signs. In our case all the velocities and acceleration are positive, because they are directed along our "positive" direction.

In the case of uniform acceleration, the average velocity can be calculated as:

$$V_{average} = \frac{V_{initial}+V_{final}}{2} = \frac{V_{initial}+V_{initial}+at}{2} = \frac{2 \cdot V_{initial}+at}{2} = V_{initial} + \frac{at}{2} \qquad (2)$$

Now, to calculate the displacement *D* we have just to multiply the average speed by the time:

$$\vec{D} = \overrightarrow{V_{average}} \cdot t = \left(\overrightarrow{V_{initial}} + \frac{\vec{a}t}{2}\right) \cdot t = \vec{V}_{initial} \cdot t + \frac{\vec{a} \cdot t \cdot t}{2} = \vec{V}_{initial} \cdot t + \frac{\vec{a} \cdot t^2}{2} \qquad (3)$$

For a negative acceleration (if the car stops, i.e. the acceleration is directed opposite to the velocity) we have:

$$D = V_{initial} \cdot t - \frac{a \cdot t^2}{2} \quad (4)$$

Now we have another way to calculate the displacement of a uniformly accelerated object. Instead of calculating the average velocity and multiplying it by the time we can use Formula 2 and calculate the displacement directly using initial velocity, time and acceleration.

Graphic representation of the displacement at uniformly accelerated motion.

Let us start with a simple case of uniform motion. "Uniform" means that the velocity does not change with time. The plot "velocity vs. time" is shown in Figure 2.

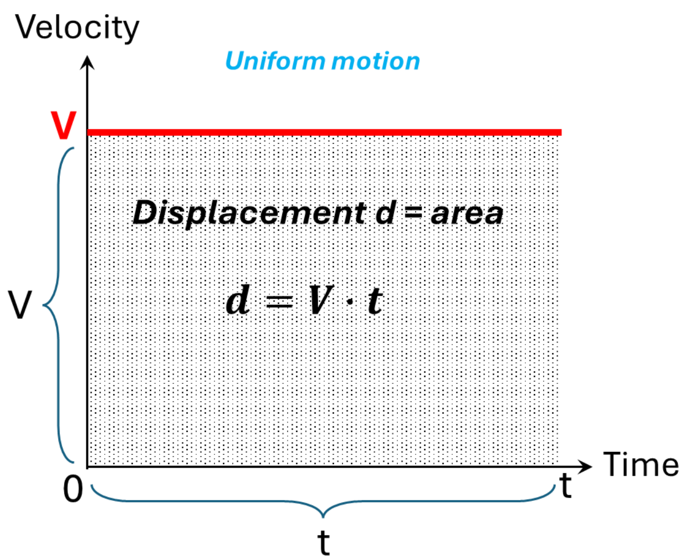


Figure 2.

The displacement value D can be calculated as:

$$D = V \cdot t \quad (5).$$

We can see that the displacement value is equal to the area of rectangle formed by the coordinate axes and the velocity.

Now consider the case of uniform acceleration. "Uniform acceleration" means that the acceleration does not change with time, but the velocity V changes with time according to expression (1). First, assume that the initial velocity is zero. The graph "velocity vs time" is shown in Figure 3 a. In this case, the displacement value can be calculated as

$$D = \frac{a \cdot t^2}{2} \qquad (6)$$

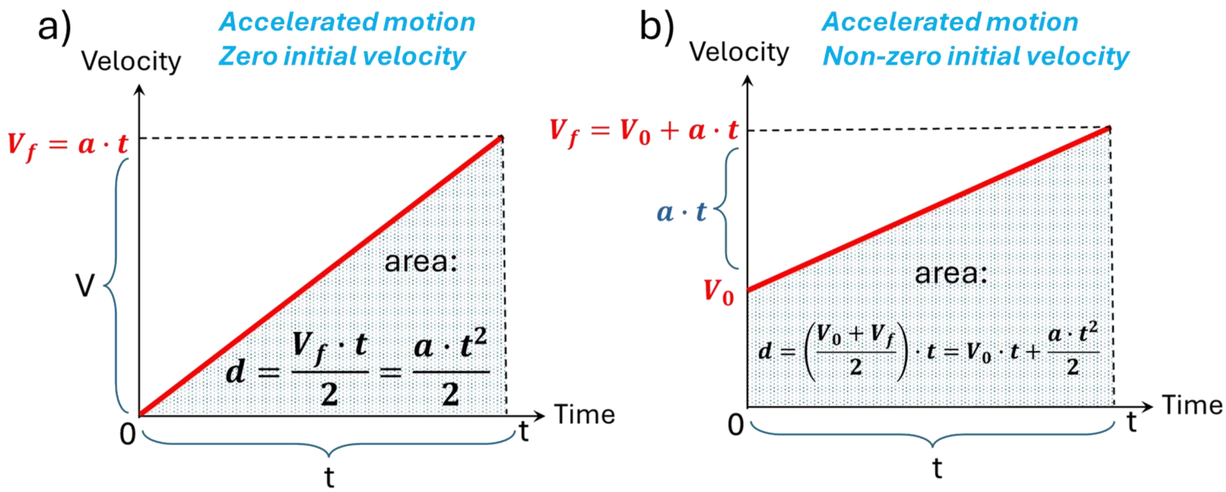


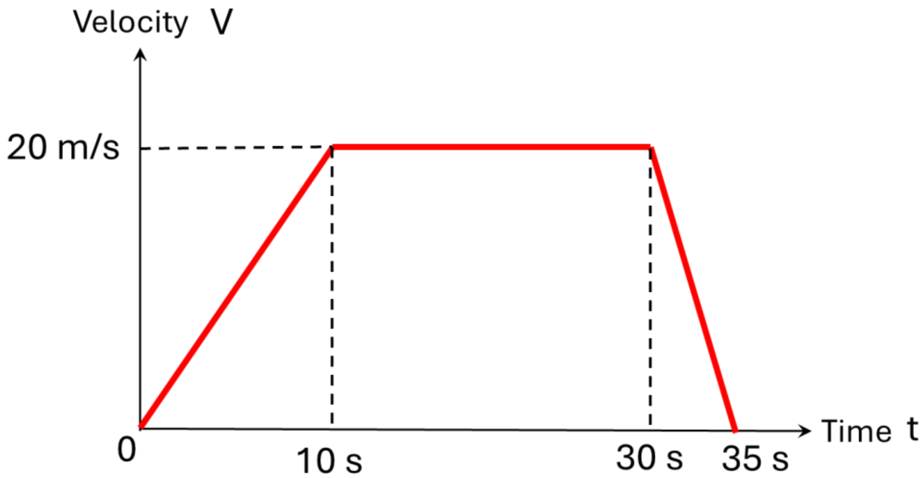
Figure 3.

And again, the displacement is equal to the area of triangle, formed by the x axis and the velocity plot. If we flip this triangle 90º clockwise, the velocity V represents the length of the base, and the time represents the height of the triangle. If the object has non-zero initial velocity V_0, the time dependence of the velocity is shown in Figure 3 (b). Now the velocity plot and the axes form geometrical figure which is called “trapezoid”. Trapezoid has at least 2 parallel sides which are called “the bases”. The area of the trapezoid can be calculated as half of the sum of the bases, multiplied by the height. In our case, the bases are $\boldsymbol{V_0}$ and $\boldsymbol{V_0 + a \cdot t}$. The heigh is time $\boldsymbol{t}$. So, for the area we have, as expected:

$$Area = D = \frac{V_0 + (V_0 + a \cdot t)}{2} \cdot t = V_0 \cdot t + \frac{a \cdot t^2}{2} \qquad (6)$$

Problems:

1. A boy staying at the roof of a building throws tennis ball vertically up at a speed of 20m/s. Find its displacement in 6 seconds after the start. Make a plot of velocity vs time.
2. The velocity of a car as function of time is plotted below. Find the displacement of the car.



3. A coin has been falling for 3 sec. An initial velocity of the coin was 0. Find the displacement of the coin during the third second.