Homework 2

Speed and velocity. Average speed and average velocity.

We started discussing speed and velocity. ***Speed*** is a *distance* passed per unit time. To find speed we have to divide the distance to the time, which required to pass the distance.

$$speed = \frac{distance}{time}\ , \text{or}$$

$$s = \frac{d}{t}$$

Speed is a scalar. It means that it is just a number and has no direction. As we mention speed the direction of the motion is not important for us. For example, a speed limit sign specifies the maximum speed independently on the direction of your motion. In contrast to speed, the velocity is a vector. The ***velocity*** is a *displacement* per unit time.

$$\overrightarrow{velocity} = \frac{\overrightarrow{displacement}}{time}\ , \text{or}$$

$$\vec{v} = \frac{\vec{d}}{t}$$

The arrows over the characters show that the corresponding parameters are vectors. How can we multiply or divide a vector by a number? How does it work and what we will obtain as the result, vector or scalar? The answer is simple: multiplying vector by a positive number, we just change the vector's length while the vector's direction stays intact. For example, multiplying a velocity vector by 2, we increase the length of the vector, i.e. the speed 2 times, by the direction of motion stays the same. If we divide by 2, the vector becomes twice shorter (see Figure 1 below):

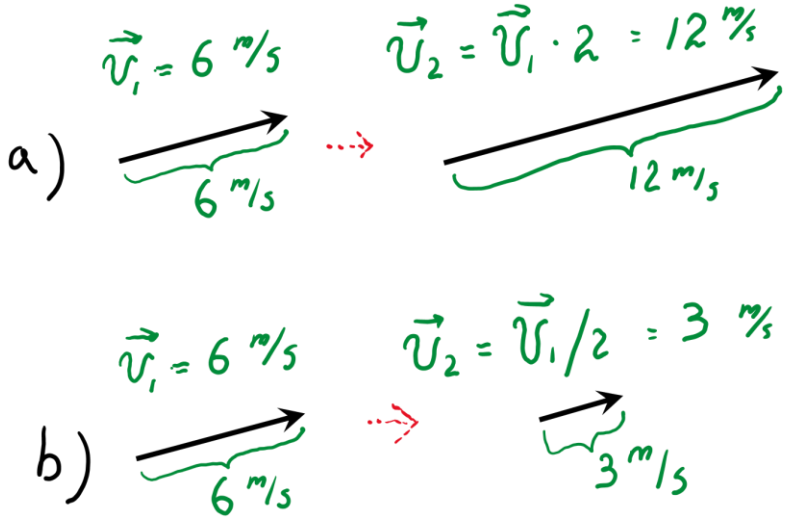


Figure 1.

If we multiply a vector by a negative number, in addition to change of the vector's length, its direction is changed to the opposite (Figure 2).

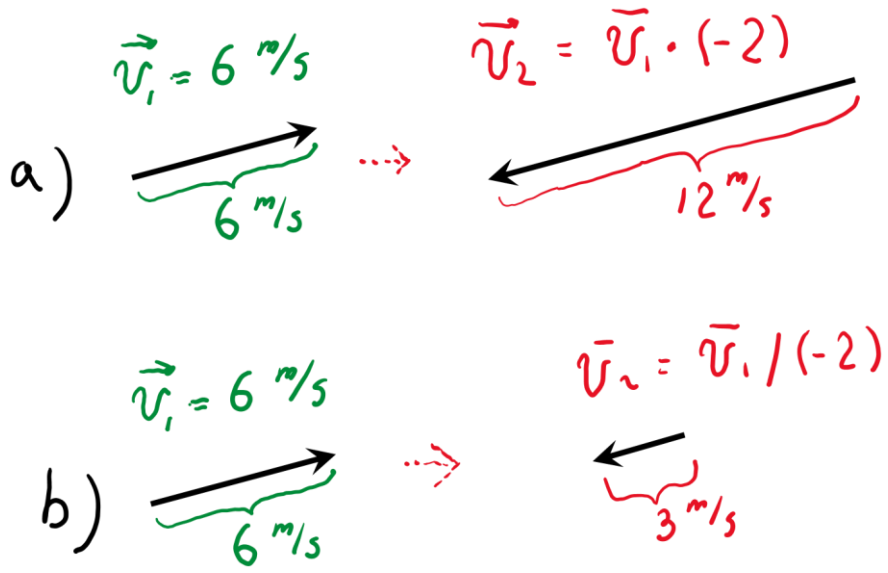
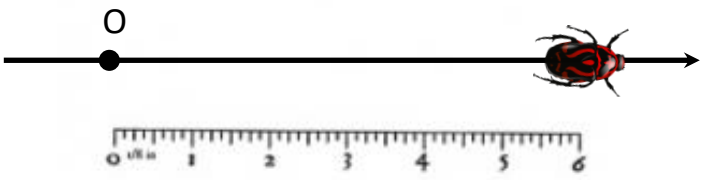


Figure 2.

If the velocity of the object does not change, the motion is called *uniform*. Since the velocity is a vector, it has, as we already know, both magnitude and direction. If the velocity does not change, both speed and direction of the motion remain constant (which means that they do not change as well). So, uniform motion is motion along a straight line. Motion along a straight line is called *rectilinear motion*. Although any uniform motion is rectilinear, rectilinear motion is not necessarily uniform.

This year we will be mostly studying rectilinear motion. To specify the position of the object moving along a straight line we need a reference point. It is convenient to choose a point at the line of motion and calculate all the distances and displacements with respect to this point. We will call this point "origin" and mark with the character O.



The direction from the origin to our right we will call positive. The opposite direction is negative. This choice is arbitrary. You may choose the positive direction as you wish.

Positive or negative sign of the velocity or displacement will just indicate the *direction* of the motion. In contrast, speed and distance cannot be negative – they have no direction.

Average speed and velocity.

Most of the motions around us are non-uniform. It means that the speed and /or velocity are changing during the motion. In this case we can introduce *average speed* and *average velocity*. Average speed is a rate of *total* distance and time interval required to cover this distance.

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

For example, you have to go for 1km. First you run, then stop for a while to take a break and, finally, you walk. It took 15 minutes to cover 1 km. The average speed in this case is

$$\text{average speed} = \frac{\text{total distance} = 1\text{km} = 1000\text{m}}{\text{total time} = 15\text{min} = 15 \text{ x } 60\text{s} = 900\text{s}} \approx 1.11 \frac{m}{s}$$

It means that instead of running, taking a rest and, finally, walking you just keep going with a uniform speed of 1,11m/s you will pass 1 km for the same time of 15min.

Average velocity is a rate of total ***displacement*** and time interval required to complete this displacement.

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

For example, if at the end of a very long trip you returned to the starting point, your average velocity is zero, because your displacement is zero.

Problems:

1. A student walks 120 m to the library in 2 min, stops there for 5 min, and then walks 180 m back home in 3 min.

   (a) Find the **average speed** during the entire trip (including the stop).

   (b) Find the **average velocity** (magnitude and direction).

2. A car passed 30km at the speed of 15m/s. Then the car turned back and spent 1 hour to pass 40 km. Find average speed and average velocity of the car? Make a picture.
3. A person walks from point A to point B (2 km apart) at 3 km/h and returns along the same path at 6 km/h.

   (a) Compute the **average speed** for the entire journey.

   (b) Compute the **average velocity**.

   (c) Explain why the answer for average speed is **not** the simple arithmetic average of 3 km/h and 6 km/h.

4. A particle moves on a straight line according to this velocity–time graph:

   For the first 4 s: velocity = +5 m/s

   For the next 3 s: velocity = −2 m/s

   For the last 3 s: velocity = +1 m/s

   (a) Draw the displacement–time graph.

   (b) Find the **net displacement** after 10 s.

   (c) Find the **average velocity** and **average speed**.