

## Homework 13

### Impulse

From the second Newton's law we know that we need to apply force to change velocity of an object. Let us consider rectilinear motion (motion along a straight line) of an object with uniform acceleration  $\mathbf{a}$  due to constant force  $\mathbf{F}$ :

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} \quad (1)$$

Where  $m$  is the mass of the object (we assume that the mass does not change). Let us calculate the change in the object's velocity during the time  $t$ :

$$\vec{V}_{final} - \vec{V}_{initial} = \vec{\mathbf{a}} \cdot t = \frac{\vec{\mathbf{F}}}{m} \cdot t \quad (2)$$

Let us multiply both left and right parts of the equation 2 by the mass  $m$ :

$$m\vec{V}_{final} - m\vec{V}_{initial} = \vec{\mathbf{F}} \cdot t \quad (3)$$

We also know that product of mass and velocity is momentum. So we can write:

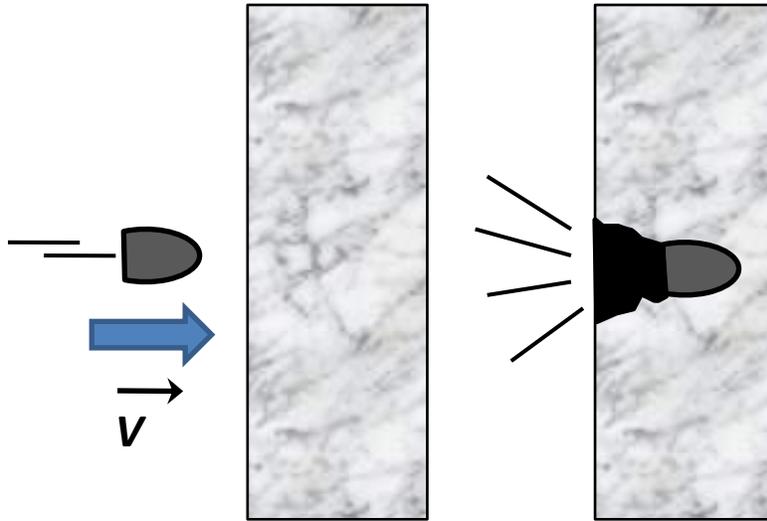
$$\vec{P}_{final} - \vec{P}_{initial} = \vec{\mathbf{F}} \cdot t \quad (4)$$

If the force  $\mathbf{F}$  applied during the time  $t$ , the product  $\mathbf{F}t$  is called **impulse**. Impulse is a vector. The change of the object momentum is equal to the impulse applied to the object.

We can divide both parts of the formula (4) by the time  $t$ . We will have:

$$\frac{\vec{P}_{final} - \vec{P}_{initial}}{t} = \vec{\mathbf{F}} \quad (5)$$

Looking at formula (5) we can say that the rate of an object's momentum change is equal to the force applied to the object. To understand better formulae (4) and (5) let us consider a bullet hitting a wall.



The bullet has relatively low mass (~10g) but it moves at a high velocity (~300-400m/s). We can calculate the momentum of the bullet:  $P=0.010\text{kg}\cdot 400\text{m/s}=4\text{kgm/s}$ . Imagine that the bullet hits a wall and stops in 0.01second. Let us calculate the force applied to the bullet by the wall (I choose the direction of the bullet motion as positive):

$$1) P_{final} - P_{initial} = 0 - 4\text{kgm/s} = F \cdot t = F \cdot 0.01\text{s}$$

$$2) -4\text{kgm/s} = F \cdot 0.01\text{s}$$

$$3) F = -4\text{kgm/s} \div 0.01\text{s} = -400\text{N}$$

The force is negative – this means that it is directed against the direction of the bullet motion. But from the third Newton's law we know that the force applied by the bullet to the wall has same magnitude but opposite direction. So the answer is 400N.

Problems:

1. A rubber ball ( $m=10\text{g}$ ) is dropped down from the top of 180m building. The ball hits the ground and bounces up with the same speed. Find the force applied by the ball to the ground if the collision time is 0.01s.
2. A soccer ball with  $m=400\text{g}$  moves at a speed of 25m/s. If the ball will hit the chest of the goalkeeper it bounces back with the same speed and the collision time is 0.025s. If the goalkeeper will catch the ball with his (her) hands, the speed of the ball becomes zero in 0.04s. Find the force applied by the ball to the goalkeeper in both cases.