Homework 13.

Momentum

Let us consider two balls of masses $\boldsymbol{m_1}$ amd $\boldsymbol{m_2}$, moving toward each other and then colliding (Figure 1 a,b). After collision, each ball changes the direction of motion to the opposite one (Figure 1 c). In the moment of collision each ball exerts force to the other ball. These forces, according to the 3rd Newton's law are equal in absolute value but have opposite directions (Figure 3,b). Imagine that we know the velocities $\boldsymbol{V_{i1}}$ and $\boldsymbol{V_{i2}}$ of the balls before the collision and velocity of the first ball after the collision $\boldsymbol{V_{f1}}$. Can we calculate the velocity of the second ball $\boldsymbol{V_{f2}}$ after the collision?

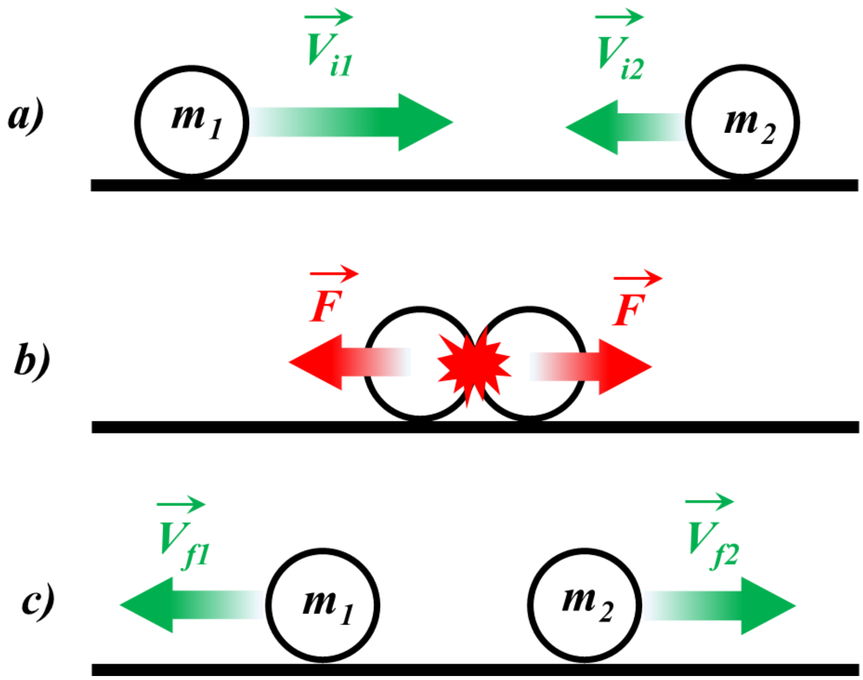


Figure 1.

To do that, let us calculate the acceleration of the first ball:

$$\overrightarrow{a_1} = \frac{\vec{V}_{f1} - \vec{V}_{i1}}{t} \qquad (1),$$

Here $\boldsymbol{t}$ is the collision time, which is very short and the sane for both balls. This acceleration, according to the 2nd Newton's law is equal to:

$$\frac{\vec{V}_{f1} - \vec{V}_{i1}}{t} = \frac{\vec{F}}{m_1} \qquad (2)$$

Then we can multiply both parts of equation (2) to $\boldsymbol{t}$ and to $\boldsymbol{m_1}$. The result is below:

$$\boldsymbol{m_1 \cdot \vec{V}_{f1} - m_1 \cdot \vec{V}_{i1} = \vec{F} \cdot t} \qquad (3)$$

We can also do the same with ball 2 and obtain:

$$\boldsymbol{m_2 \cdot \vec{V}_{f2} - m_2 \cdot \vec{V}_{i2} = -\vec{F} \cdot t} \qquad (4)$$

The “minus” sign in the right part of equation (4) is because the force applied to the second ball is opposite to the force, applied to the first ball. Now we can add equation (3) and equation (4). The right parts of both equations cancel each other and we have:

$$\boldsymbol{m_1 \cdot \vec{V}_{f1} - m_1 \cdot \vec{V}_{i1} + m_2 \cdot \vec{V}_{f2} - m_2 \cdot \vec{V}_{i2} = 0} \qquad \text{(5), or}$$

$$\underbrace{\boldsymbol{m_1 \cdot \vec{V}_{i1} + m_2 \cdot \vec{V}_{i2}}}_{\substack{\text{Total momentum} \\ \text{before collision}}} = \underbrace{\boldsymbol{m_1 \cdot \vec{V}_{f1} + m_2 \cdot \vec{V}_{f2}}}_{\substack{\text{Total momentum} \\ \text{after collision}}} \qquad (6)$$

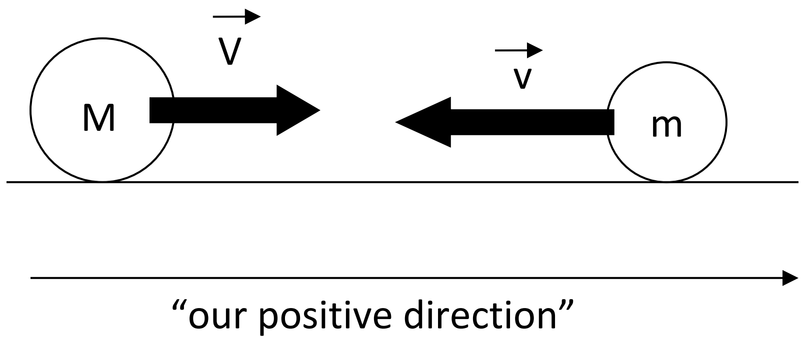
So, as we can see, the sum of the velocities of the balls multiplied by their masses in the same before and after collision remains the same. The product of the mass and velocity of an object is called ***momentum***. Equation (6) expresses on of the most important laws of mechanics *momentum conservation*. Momentum is a vector: it has both direction and magnitude.

$$\vec{P} = m\vec{V}$$

Momentum is measured in *kg m/s*. As we can see, we can change the momentum by changing the velocity or the mass of an object. To change the velocity we need to apply force. To change the mass of a moving object we could, for example, put an additional load to the rolling cart.

As we can see, if no forces applied to an object (or objects) and total mass of the object (objects) does not change, the momentum does not change as well.

If an object or a group of objects are isolated (which means that there is no external force applied to them) the total momentum of this object or group of objects does not change no matter how strong they interact with each other. In this case we can say that the momentum of the objects *conserves*.



Example: Find total net momentum of two balls with masses *m* and *M* and velocities V and v rolling toward each other (Figure 2).

Solution: First we will choose "positive direction". The momentum and velocity of an object have same direction. I picked up "right-to-left", but the result (as we know) will not change if we choose "left-to-right". The momentum of the left ball is: P=MV. We consider it as positive, because the momentum (as the velocity) "looks" in our "positive" direction. The momentum of the left ball p= - mv. It is negative, because it "looks" in the opposite direction. So the total net momentum, ***P*** is:

$$P = MV - mv$$

If MV is larger than mv, the total net momentum is positive. It means that it is directed "positively" - left-to-right. If MV is less than mv the total net momentum is negative and "looks" right-to-left.

Problems:

1. Two cars with mass 1000kg and 2000kg go toward each other. The speed of the first car is 50km/h, the speed of the second is 40 km/h. Find the total net momentum of the two cars. Make a picture.

2. A 80kg jogger runs with a constant acceleration of 1/5 m/s^2 for 10 seconds. How did his momentum change during this time?

4. A 10kg ball moving to the right at a velocity of 10m/s hits a 5kg ball which was at rest before the collision. After the collision the smaller ball starts moving to the right at a velocity10m/s. Find the velocity of the heavy ball after the collision.

5. A fox is chasing a small rabbit. The momentum of the fox is equal to the momentum of the rabbit. Will the fox catch the rabbit?

6. You send a 100g ball up and it returns back in 6 seconds. Find the initial momentum of the ball and its momentum in the highest point.