Homework 4.

Average speed and velocity. Acceleration.

Most of the motions around us are non-uniform. It means that the speed and /or velocity are changing during the motion. In this case we can introduce *average speed* and *average velocity*. Average speed is a rate of *total* distance and time interval required to cover this distance.

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

For example, you have to go for 1km. First you run, then stop for a while to take a break and, finally, you walk. It took 15 minutes to cover 1 km. The average speed in this case is

$$\text{average speed} = \frac{\text{total distance} = 1\text{km} = 1000\text{m}}{\text{total time} = 15\text{min} = 15 \text{ x } 60\text{s} = 900\text{s}} \approx 1.11\frac{m}{s}$$

It means that instead of running, taking a rest and, finally, walking you just keep going with a uniform speed of 1,11m/s you will pass 1 km for the same time of 15min.

Average velocity is a rate of total ***displacement*** and time interval required to complete this displacement.

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

For example, if at the end of a very long trip you returned to the starting point, your average velocity is zero, because your displacement is zero.

Last time we also discussed *acceleration*. In everyday life we use the word *acceleration* to describe increase of the speed of a moving object. Acceleration in physics has different meaning. It is change in *velocity* per unit time. If a uniformly accelerated object increased its velocity from initial velocity $\overrightarrow{V_0}$ to final velocity $\overrightarrow{V_f}$ in time period t, we can calculate the acceleration $\vec{a}$ as:

$$\vec{a} = \frac{\overrightarrow{V_0} - \overrightarrow{V_f}}{t} \qquad (1)$$

Any time the speed and/or the direction of motion of an object changes we deal with *accelerated* motion. An example of acceleration motion is falling. We know that any object falls down with acceleration of ~10m/s^2 (9.8 m/s^2, to be exact).

Acceleration is a vector – it has both magnitude and direction.

For the case of rectilinear motion (just to remind – this is the motion along a straight line) there are two major cases:

1. The object speed increases with time:

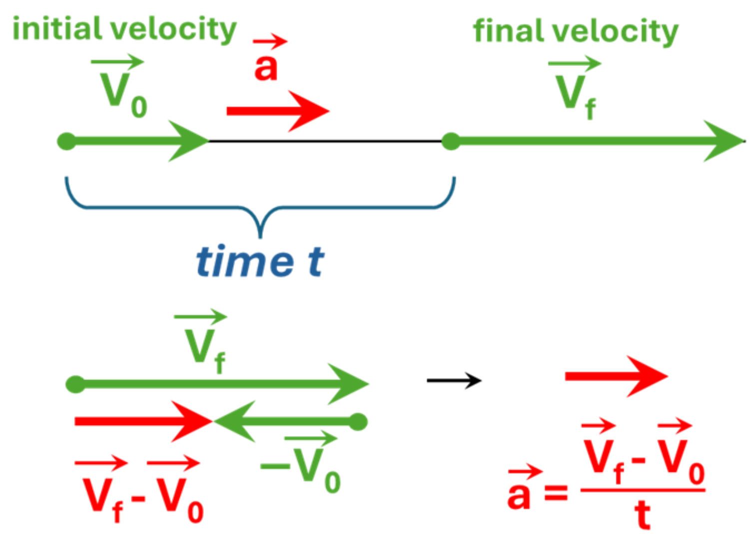

Figure 1. Accelerated motion: speed increases with time

In this case the velocity and acceleration have same sign and speed of the object is *increasing* with time. The acceleration magnitude gives us the rate of the speed increase. For example acceleration of 5meters per second per second (this is not a typo!) means that the speed increases for 5m/s every second. It is usually denoted as 5m/s^2 (five meter per second square)

2. The object speed decreases with time:

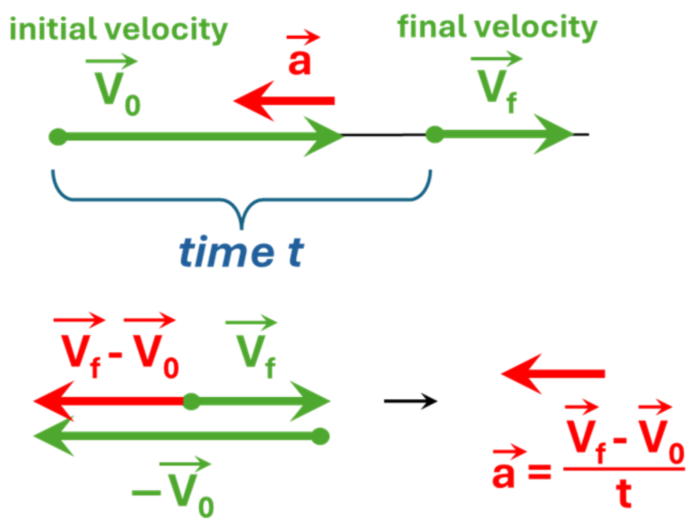

Figure 1. Accelerated motion: speed decreases with time

In this case the velocity and acceleration have opposite signs and speed of the object is *decreasing* with time. The acceleration magnitude gives us the rate of the speed decrease. For example, acceleration of -5meters per second per second means that the speed decreases for 5m/s every second.

For some complicated types of motion (oscillations of a pendulum, for example) acceleration changes with time. We will study only the motion at a constant acceleration ("constant" means "does not change") . If we know acceleration and initial velocity we can easily find from expression (1) the velocity at any later moment:

$$\vec{V} = \vec{V}_0 + \vec{a} \cdot t \qquad (2)$$

Velocity after the time t = ***Initial velocity*** plus ***Acceleration*** multiplied by the ***time***

For example, if you just let a pebble go down, the initial velocity is zero. But you can throw the pebble down. In this case the pebble starts accelerating from nonzero velocity.

Just to remind, arrows on top of some characters in formula (1) mean that the corresponding physical parameters are vectors. When you will be solving problems, after you chose the "positive" direction you will be able to put correct signs before $\boldsymbol{V}$, $\boldsymbol{V_0}$ and $\boldsymbol{a}$. After the signs are chosen you can consider these parameters as regular numbers and you do not need to use the arrows anymore.

Problems:

1. Imagine that you dropped a penny from Empire State Building (please, never do it in real life!). Calculate the speed of the coin in 5 seconds. (Just to remind: acceleration due to gravity is 9.8 m/s^2).

2. The bullet leaves the barrel and starts moving up at a speed of 400m/s. In what time the bullet will stop?

3. A ball is thrown straight down from the height of 10m and hits the ground in 1 second. Find velocities of the ball in the beginning and at the end of the motion.

   (*Hint: in case of a uniformly accelerated motion, average velocity can also be calculated as an arithmetic mean of initial and final velocities*).