Homework 21.

Electrostatic potential energy

While the electric force between two charges $\boldsymbol{Q}$ and $\boldsymbol{q}$ can be expressed as:

$$\left|\vec{F}\right| = k\frac{Q\cdot q}{r^2}\cdot \quad (1),$$

where r is the distance between the charges, the potential energy of the charges is

$$E_P = k\frac{Q\cdot q}{r}\cdot \quad (2).$$

Note that it is just r rather than r^2 in the denominator.
How we can obtain this formula?
Let us take two small charged objects – "point charges", with charges $\boldsymbol{Q}$ and $\boldsymbol{q}$, both being positive and separated by a distance $\boldsymbol{r_1}$ (Figure 1).

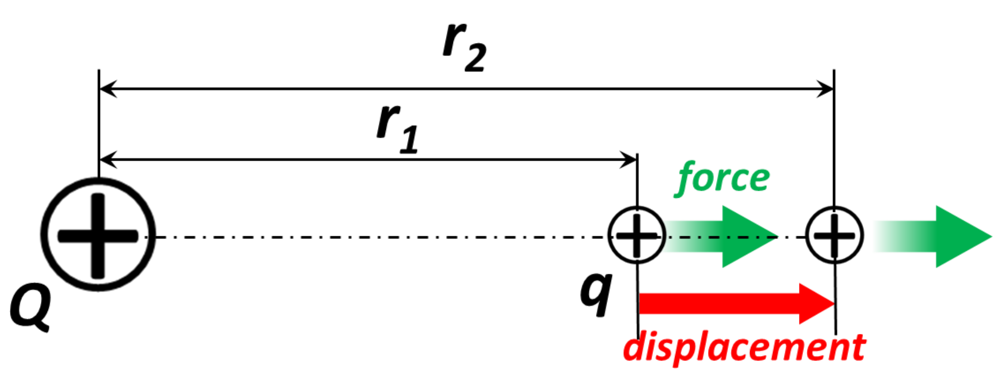


Figure 1.

Initially, the positions of both charges are fixed, the charges have some potential energy and their kinetic energy is zero. Then we release charge $\boldsymbol{q}$. This charge, being repelled by charge $\boldsymbol{Q}$, starts moving. The distance, separating the charges, increases from $\boldsymbol{r_1}$ to $\boldsymbol{r_2}$ and kinetic energy of charge $\boldsymbol{q}$ increases. According to work-energy theorem, the change in kinetic energy is equal to the work, done by Coulomb force on charge $\boldsymbol{q}$. On the other hand, the total energy of charge $\boldsymbol{q}$ stays the same. (The position of charge Q stays fixed - we are "holding it" – so its kinetic energy is still zero). So, the work W, i.e. the increase of the kinetic energy ($\boldsymbol{E_{K2}}$-$\boldsymbol{E_{K1}}$) is equal to the decrease of the potential energy ($\boldsymbol{E_{P1}}$-$\boldsymbol{E_{P2}}$):

$$E_{P1} - E_{P2} = work = force \cdot displacement\ along\ the\ force$$

$$E_{P1} - E_{P2} = k\frac{Q\cdot q}{r^2}\cdot(r_2 - r_1) \quad (3)$$

But there is one difficulty: the Coulomb force depends on the distance r between the charges and this distance changes from r_1 to r_2 as charge q moves away ☹. Which number I should plug instead of r in the denominator of the expression for the force?
So, what I suggest:

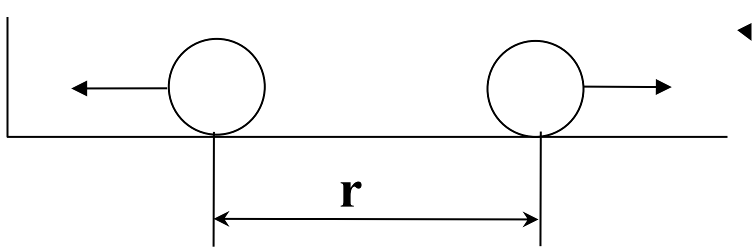
1. Let us assume that the displacement of the charge q is very small, so $r_1 \approx r_2$
2. Let is replace r^2 in the denominator of expression (3) to $r_1 r_2$. We have to use square of the ***average*** distance but r_1 is a bit less than the average distance and r_2 is a bit higher than the average distance, so his replacement seems to be OK:

$$E_{P1} - E_{P2} = k\frac{Q \cdot q}{r_1 \cdot r_2} \cdot (r_2 - r_1) = kQq \cdot \left(\frac{r_2}{r_1 \cdot r_2} - \frac{r_1}{r_1 \cdot r_2}\right)$$

$$E_{P1} - E_{P2} = k\frac{Q \cdot q}{r_1} - k\frac{Q \cdot q}{r_2} \quad (4)$$

So, finally we came to formula (2) for electrostatic potential energy.

If two charges are of different signs than the potential energy is *negative*. It just means that the closer two charges are, the less is their potential energy. The charges with the opposite signs "like" to be as close as possible to reduce their potential energy. So, negative electrostatic potential energy means attraction. For two charges of same sign, the potential energy is *positive* and increases with the distance between them. It means repulsion.
Here is an example:
*Problem: Imagine that we have two identical negatively charged balls (the mass is **M** charge is **q**) are separated by the distance r_{before}. We let the balls go and they start moving. What are the velocities **v** of the balls when the distance between them is r_{after}?*



Solution:
We can try to use Coulomb's law to calculate the force applied to each ball, find acceleration of each ball and, using kinematics formula calculate the time and final velocities. This is a long way, and, moreover, soon we will meet a serious difficulty – the interaction force and acceleration change with the distance.
There is another, much simpler solution which is based on the energy conservation law. As the balls move away from each other their potential energy decreases, but the kinetic energy of both ball increases. Total energy conserves so the increase of kinetic energy equals to decrease of potential energy. Since the balls are identical, each of them gets halve of the total kinetic energy (symmetry consideration):

$$k\frac{q\cdot q}{r_{after}}+2M\frac{v^2}{2}$$ - total energy when the distance is 20m

(!) ***we do not multiply the potential energy by 2 since this is "joint" energy of the system of 2 charges. However, we multiply the kinetic energy by 2 since M is the mass of one ball.***

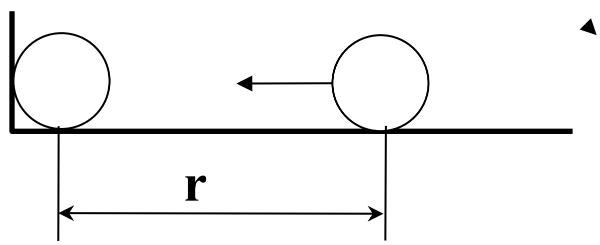
$$k\frac{q\cdot q}{r_{before}}$$ - total energy in the beginning, when the distance is 10m

$$k\frac{q\cdot q}{r_{after}}+2M\frac{v^2}{2}=k\frac{q\cdot q}{r_{before}}$$ - energy conservation

In this equation we know everything except *v* – so we can easily calculate it.

The homework problems are below:

1. A 2kg positively charged (the charge is 0.001C) ball is pushed toward identical ball with the same charge. The position of the second ball is fixed. When the distance between the balls is 10m, the speed of the first ball is 1m/s. Find the distance at which the moving ball stops?



2. 3 positively charged balls 100g each are places in the corners of an equilateral triangle with the side of 1m. The charge of each ball is 0.001C. After we let them go, the balls will move away from each other and, in a certain time the side of the triangle formed by the balls will be 10m Find the velocities of the balls in this moment.

3. Two metal balls with masses 2kg and 1 kg, are charged. The charge of the first ball is –0.1C, the charge of the second is +0.3C. The balls are separated by a distance of 5m. After the balls are released, they start moving toward each other. Find the velocities of the balls when the distance between them is 2m. (Hint use both energy and momentum conservation laws)