Homework 15

**Graphical representation of the work done by the ideal gas.**

During the recent class we discussed graphic representation of the ideal gas processes:

Imagine cylinder with a piston, filled with ideal gas. The gas has volume $V_1$ and pressure $P_1$. (Figure 1). We know, that for an ideal gas

$$\frac{P \cdot V}{T} = const \qquad (1)$$

“Const” means “does not change”, so we can assume that the right part of equation 1 is a certain fixed number. We do not know yet how to calculate this number for a given amount of gas, but, generally, it is possible. So, if you know any 2 of three variables P, V, T, then one can find the third one. So any 2 variables, say, P and V determine the stat of the gas. So, point 1 in Figure 1 represents initial state of the gas.

Then we increased the volume of the gas (we pull out the piston), but maintained constant pressure during the process:

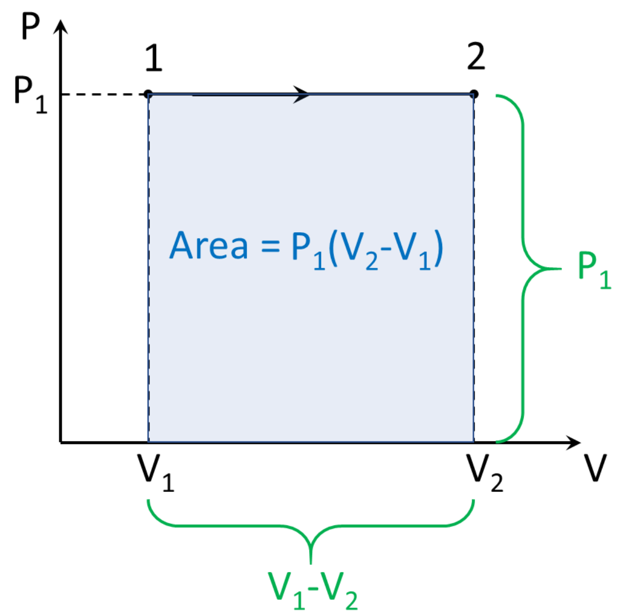


Figure 1.

At the plot, shown in Figure 1, point 1 represents the initial state of the gas, point 2 represents the final state. It is given that the gas pressure is maintained constant, so we have to increase the temperature of the gas as we are increasing the volume. So the heating process is shown as a horizontal straight line connecting points 1 and 2.

We know that as long as the gas pressure is non-zero and the volume of the gas is changing that the work is done. As the pressure does not change, the work $\Delta W$ can be calculated as pressure $P_1$ multiplied by the change of the gas volume: $\Delta W = P_1 (V_2 - V_1) = P_1 \Delta V$.

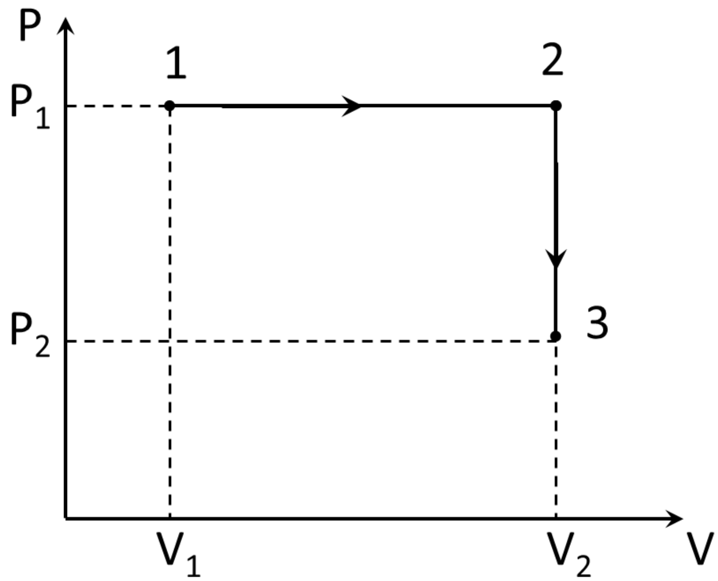


Figure 2.

Work done by the gas can be conveniently shown as the area under the line 1-2 (Figure 1). Now being in state "2" let us decrease the pressure to $P_2$ while maintaining constant volume $V_2$. We can do that by reducing the temperature. This process is shown as line 2-3 in Figure 2. The gas does not do work since the volume of the gas in this process does not change.

Then we can compress the gas while maintaining the constant pressure $P_2$ and reduce the volume to $V_1$ again. Normally, when we compress the gas we can expect the pressure to increase. But we can maintain constant pressure by cooling the gas while compressing it. This is the process 3-4 in Figure 3:

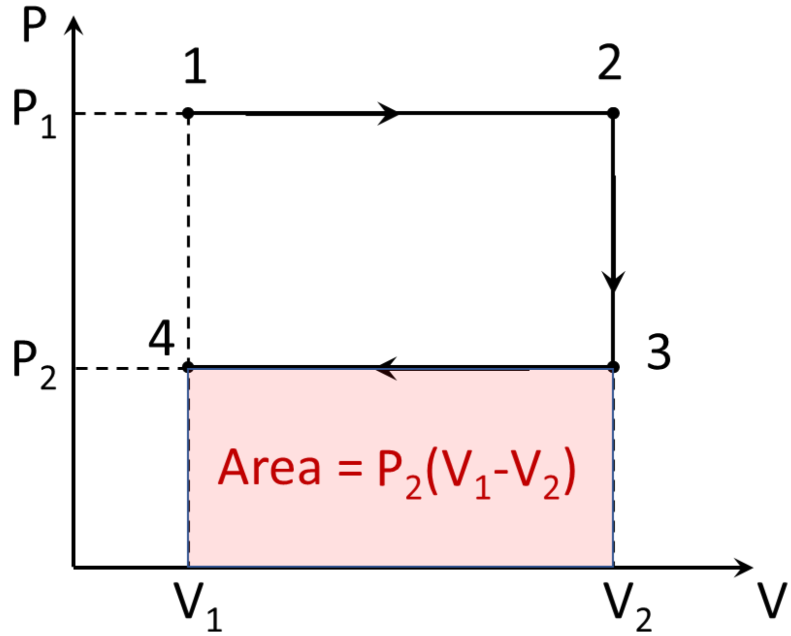


Figure 3.

During process 3-4 we do work *on the gas* by compressing it. Please notice that the change of the volume is negative: the destination volume $V_1$ is lower than the initial volume $V_2$. So we can write the work $\Delta W_{34}$ of the gas during process 3-4 as negative:

$$\Delta W_{34} = -P_2 \cdot (V_2 - V_1) = -P_2 \cdot \Delta V$$

Absolute value of this work is equal to the pink-shaded area in Figure 3.

Then we will “return” to the initial point by heating the gas at constant volume $V_1$ (process 4-1 in Figure 4). So we “moved” the gas state through the states 1-2-3-4-1 and returned to the initial state. Such process is called “cyclic process” or “cycle”.

Let us calculate the total work W, done by the gas during this cycle:

$$W = \Delta W_{12} + \Delta W_{23} + \Delta W_{34} + \Delta W_{41}$$

Please note that $\Delta W_{23} = \Delta W_{41} = 0$, since the volume during these processes does not change and the work in not done. So, we have:

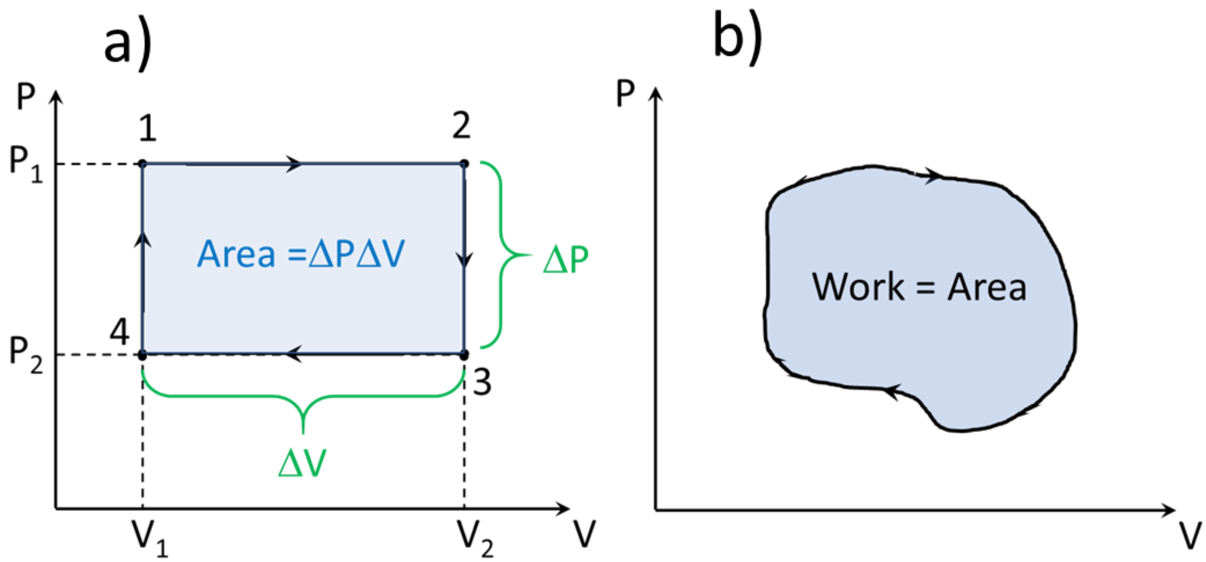


Figure 4.

$$W = \Delta W_{12} + \Delta W_{34} = P_2 \Delta V - P_1 \Delta V = \Delta P \cdot \Delta V$$

But this work is numerically equal to the area of the rectangle 1-2-3-4 (Figure 4a):

It turns out that for any closed process, work done by the gas is numerically equal to the area within the cycle process line, plotted in coordinates P-V (figure 4b)

Questions:

1. Sketch the cycle process from Figure 4a using P as a vertical axis and T as a horizontal axis.

2. Will the total work for the process shown in Figure 4a change if we “pass” it counter-clockwise: 1-4-3-2-1?