Homework 23.

Electric potential. Superposition principle.

The potential energy where of two charges separated by a distance *r* is

$$P = k\frac{q_1 \cdot q_2}{r} \quad (1)$$

Let us keep one of the charges, say, q_1 fixed and change the charge q_2. Since there is a product of the charge magnitudes in the numerator of formula (1), the potential energy will increase or decrease proportionally to the charge magnitude of q_2. We can now calculate the potential energy *per unit charge*. For this we will divide the potential energy of the interacting charges q_1 and q_2 by the magnitude of q_2:

$$\frac{P}{q_2} = k\frac{q_1 \cdot q_2}{r} \div q_2 = k\frac{q_1}{r} \quad (2)$$

We can imagine that each point of space around the charge q_1 can be characterized by the potential energy of a positive unit charge in this point. The electrostatic potential energy of a positive unit charge in a certain point is called "***electric potential***" in this point. The electric potential is a scalar.

The beauty and convenience of the concept of electric potential is that using the electrical potential we can easily calculate the potential energy of a charged object in the electric field created by arbitrary configuration of other charged objects.

Let us consider the following example.

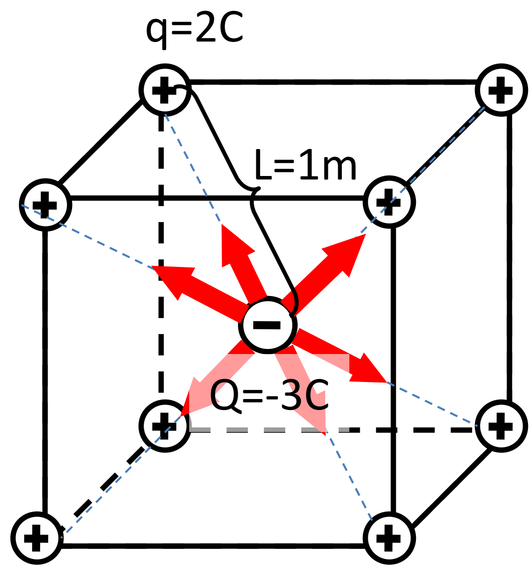

Figure 1. The red arrows show the electrostatic forces applied to the negative charge.

Given:

Eight point charges q=2C each are placed in the corners (vertices) of a cube (see Figure 1 above). The positions of the positive charges are fixed. The distance between the center and a corner of the cube is 1m. A negative charge of Q=3C is placed in the center of the cube.

Find electrostatic potential energy of the negative charge.

Solution:

As we remember, a possible way to calculate the electrostatic potential energy of a charge in a certain point is to calculate the electrostatic potential in this point and multiply it by the charge. Let us calculate the electrostatic potential in the center of the cube (black point in the Figure 2.

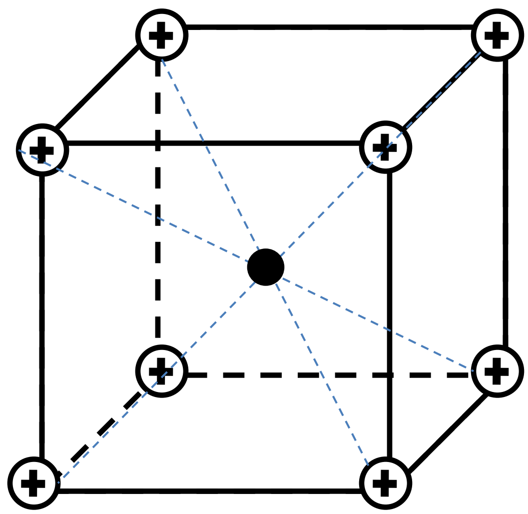

Figure 2.

At a first glance the problem looks difficult. But, in fact, it is not. To solve it, we will use the ***principle of superposition.*** According to this principle, we can calculate the potentials created in the center of the cube by each of the positive charges separately. After that we will just add these potentials together.

a) Let us pick just one positive charge (Figure 3).

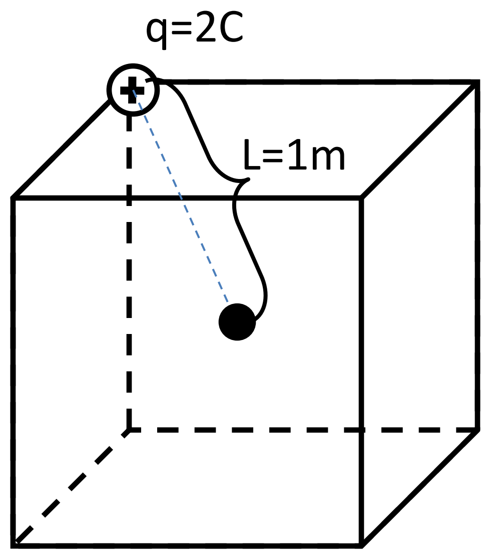

Figure 3.

Then let us calculate the electrostatic potential created by this charge at the center of the cube.

$$\boldsymbol{\varphi_{one\ charge}} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{L}} \approx 8.9 \cdot 10^9 \left(\frac{N \cdot m^2}{C^2}\right) \cdot \frac{2(C)}{1(m)} \approx 1.78 \cdot 10^8 (V)$$

b) Now, we have to take another positive charge, calculate its contribution to the potential etc. But, there is a simpler way. We can use the ***symmetry principle*** which we discussed earlier. As long as all the corners of the cube occupy equivalent positions with respect to the center of the cube, there is no reason to prefer one corner to another. Thus, the contributions of the equal positive charges placed in the corners of the cube to the potential in the cube's center should be equal. So we can just multiply the contribution of one positive charge by 8 – the number of corners.

$$\boldsymbol{\varphi_{total}} = \boldsymbol{\varphi_{one\ charge}} \cdot 8 = 1.78 \cdot 10^8 (V) \cdot 8 \approx 1.42 \cdot 10^7\ (V)$$

c) Now we can easily calculate the potential energy ***P*** of the charge ***Q=-3C*** placed in the center of the cube:

$$\boldsymbol{P} = \boldsymbol{\varphi_{total}} \cdot \boldsymbol{Q} = 1.42 \cdot 10^7\ (V) \cdot (-3)(C) = -4.26 \cdot 10^7 J$$

What happens if the charges at the "bottom" vertices are negative (Figure 3)?

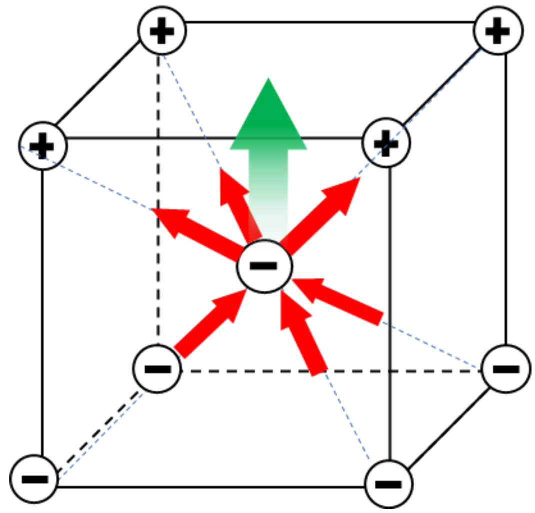

Figure 3.

Now, as we can clearly see the negative charge in the center will be pushed up, since it is repelled from the bottom and attracted to the top. So, if we let it go, it will be accelerated and, when reaches, say, the middle of the top facet it will have some kinetic energy. Based on this, we can conclude that our charge has some potential energy while placed in the center of the cube. But let us calculate this potential energy using our recipe: calculate the potential and multiply it by the charge (-3C). The contribution of each of the top positive charges we have calculated earlier: 1.78×10^8 V. We have to multiply it by 4, since there are 4 charges on the top vertices). The contribution of each of the bottom negative charges is negative: -1.78×10^8 V. We have to multiply it by 4 as well and add the positive contribution of the top charges. The result is zero! And it is correct! But if we multiply -3C by zero to find the potential energy we will get zero. No potential energy!

To resolve this paradox let us remember that not absolute potential energy, but the change of potential energy is physically meaningful. Potential energy in the center is zero, but it is not ***minimal*** potential energy. If you calculate the potential energy of our charge in the center of the upper facet, you will have a negative value. So, the change of the potential energy will be positive and will be equal to the kinetic energy acquired by our charge.

Problems

1. Find potential energy of our -3C charge in the middle of the upper face of the cube.

2. Find potential in the center of a uniformly charged thin sphere having a total charge of 1C and radius 1m.

   Hint: so far, we have been discussing only systems of point charges. The charged hollow sphere does not look like such a system. But we can "cut" the sphere to a bunch of very small pieces, each of them is similar to a point charge. As long as the total charge is *distributed evenly over the sphere surface, the charge of each piece will be proportional to the area of the piece. Then you can calculate a contribution to the center potential, made by each piece and, finally, add all the contributions.*