

HANDOUT 6

1. PRODUCT OF FACTORS OF A NUMBER

The product of all the factors of a number N is $N^{\frac{\# \text{factors}}{2}}$. Can you explain why that is?

2. CONVERTING BETWEEN NUMBER BASES

3. MATH BATTLE

1. If $x - y = 2$ and $xy = 5$, find the value of $x^3 - y^3$. (Suffolk County Math Tournament)
2. What is the remainder when $(3^{98} + 3^{99} + 3^{100})$ is divided by 5? (Suffolk Country Math Tournament)
3. In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line? (AMC 10B)
(A) 2021 (B) 2022 (C) 2023 (D) 2024 (E) 2025
4. What is $10! - 7! \cdot 6!$ (AMC 10B)
(A) -120 (B) 0 (C) 120 (D) 600 (E) 720
5. For how many integer values of x is $|2x| \leq 7\pi$ (AMC 10B)
(A) 16 (B) 17 (C) 19 (D) 20 (E) 21
6. Balls numbered 1, 2, 3, ... are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in bin B. The next 3 balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, balls numbered 22, 23, ..., 28 are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited? (AMC 10B)
(A) A (B) B (C) C (D) D (E) E
7. What is the remainder when $7^{2024} + 7^{2025} + 7^{2026}$ is divided by 19? (AMC 10B)
(A) 0 (B) 1 (C) 7 (D) 11 (E) 18
8. Let N be the product of all the positive integer divisors of 42. What is the units digit of N ? (AMC 10B)
(A) 0 (B) 2 (C) 4 (D) 6 (E) 8
9. Real numbers a , b , and c have arithmetic mean 0. The arithmetic mean of a^2 , b^2 , and c^2 is 10. What is the arithmetic mean of ab , ac , and bc ? (AMC 10B)
(A) -5 (B) $-\frac{10}{3}$ (C) $-\frac{10}{9}$ (D) 0 (E) $\frac{10}{9}$
10. Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side \overline{AD} . Let F be the intersection of lines EB and AC . What is the ratio of the area of quadrilateral $CDEF$ to the area of $\triangle CFB$? (AMC 10B)
(A) 5 : 4 (B) 4 : 3 (C) 3 : 2 (D) 5 : 3 (E) 2 : 1