

HANDOUT 5

1. MATH BATTLE AMC 10A 2024

What is the prime factorization of 2025? How many factors does it have?

It's prime factors are: 1,3,5,9,15,25,27,45,75,81,135,225,405,675 and 2025.

What is the sum of factors of 2025?

1. What is the value of $9901 \cdot 101 - 99 \cdot 10101$?
 (A) 2 (B) 20 (C) 200 (D) 202 (E) 2020
2. A model used to estimate the time it will take to hike to the top of the mountain on a trail is of the form $T = aL + bG$, where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimates it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?
 (A) 240 (B) 246 (C) 252 (D) 258 (E) 264
3. What is the sum of the digits of the smallest prime that can be written as a sum of 5 distinct primes?
 (A) 5 (B) 7 (C) 8 (D) 10 (E) 13
4. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?
 (A) 20 (B) 21 (C) 22 (D) 23 (E) 24
5. What is the least value of n such that $n!$ is a multiple of 2024?
 (A) 11 (B) 21 (C) 22 (D) 23 (E) 253
6. What is the minimum number of successive swaps of adjacent letters in the string $ABCDEF$ that are needed to change the string to $FEDCBA$? (For example, 3 swaps are required to change ABC to CBA ; one such sequence of swaps is $ABC \rightarrow BAC \rightarrow BCA \rightarrow CBA$.)
 (A) 6 (B) 10 (C) 12 (D) 15 (E) 24
7. The product of three integers is 60. What is the least possible positive sum of the three integers?
 (A) 2 (B) 3 (C) 5 (D) 6 (E) 13
8. Amy, Bomani, Charlie, and Daria work in a chocolate factory. On Monday Amy, Bomani, and Charlie started working at 1 : 00PM and were able to pack 4, 3, and 3 packages, respectively, every 3 minutes. At some later time, Daria joined the group, and Daria was able to pack 5 packages every 4 minutes. Together, they finished packing 450 packages at exactly 2 : 45PM. At what time did Daria join the group?
 (A) 1 : 25 PM (B) 1 : 35 PM (C) 1 : 45 PM (D) 1 : 55 PM (E) 2 : 05 PM
9. In how many ways can 6 juniors and 6 seniors form 3 disjoint teams of 4 people so that each team has 2 juniors and 2 seniors?
 (A) 720 (B) 1350 (C) 2700 (D) 3280 (E) 8100

10. Consider the following operation. Given a positive integer n , if n is a multiple of 3, then you replace n by $\frac{n}{3}$. If n is not a multiple of 3, then you replace n by $n + 10$. For example, beginning with $n = 4$, this procedure gives $4 \rightarrow 14 \rightarrow 24 \rightarrow 8 \rightarrow 18 \rightarrow 6 \rightarrow 2 \rightarrow 12 \rightarrow \cdots$. Suppose you start with $n = 100$. What value results if you perform this operation exactly 100 times?
- (A) 10 (B) 20 (C) 30 (D) 40 (E) 50