

HANDOUT 4

1. LCM AND GCD

To find the **GCD** of some numbers, take the prime factorization of each number and select only the primes that are common to the least power.

To find the **LCM** of some numbers, take the prime factorization of each number and select all the primes and to the largest power among the them.

Given any two numbers a, b ,

$$\mathbf{LCM(a, b) \cdot GCD(a, b) = a \cdot b}$$

2. FROBENIUS COIN PROBLEM

This problem is also known as the **Chicken McNugget Theorem** or **Postage Stamp Problem**. Frobenius was a German mathematician that posed a problem about the largest amount of money that could not have been made with certain types of coins.

For any two relatively prime numbers a, b (where $\gcd(a, b) = 1$), the greatest integer that **cannot** be written in the form $sa + tb$ is $ab - a - b$, where s, t are non negative integers.

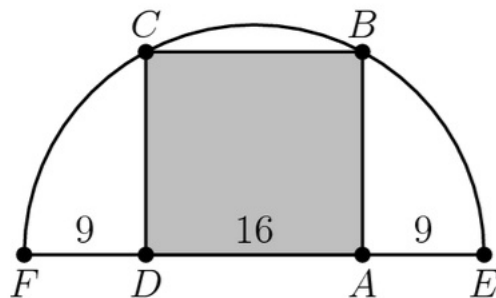
3. TEST FOR PRIME

To test whether a number N is prime or not, you have to test only with primes less than \sqrt{N}

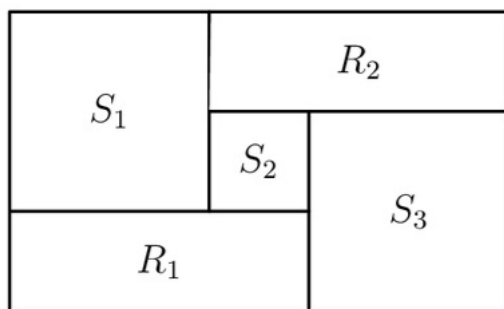
4. MATH BATTLE

1. What is the sum of factors of 5!?
2. What is the largest power of 3 that divides 9!?
3. What the largest power of 3 that divides 99!?
4. Sri and Marvin begin jogging around a track, starting at the finish line and going in the same direction. Sri completes a lap every 78 seconds, while Marvin completes a lap in 90 seconds. At the end of their jogging, they cross the finish line at the same time in a whole number of minutes for the first time. How many more laps did Sri run than Marvin?
5. McDonald's sold its nuggets in packs of 9 and 20. Find the largest number of nuggets that could not have been bought with these packs?

6. A number N is inserted into the list 2, 6, 7, 7, 28. The mean is now twice as great as the median. What is N ? (AMC 8 2025, P14)
 (A) 7 (B) 14 (C) 20 (D) 28 (E) 34
7. Five distinct integers from 1 to 10 are chosen, and five distinct integers from 11 to 20 are chosen. No two numbers differ by exactly 10. What is the sum of the ten chosen numbers? (AMC 8 2025, P 16)
 (A) 95 (B) 100 (C) 105 (D) 110 (E) 115
8. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n ? (AMC 10A 2022, P7)
 (A) 3 (B) 6 (C) 8 (D) 9 (E) 12
9. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X . The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all positive values of X ? (AMC 10A 2022, P8)
 (A) 10 (B) 26 (C) 32 (D) 36 (E) 40
10. The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet? (AMC 10B 2015, P15)
 (A) 41 (B) 47 (C) 59 (D) 61 (E) 66
11. Rectangle $ABCD$ is inscribed in a semicircle with diameter \overline{FE} , as shown in the figure. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$? (AMC 8 2020, P18)



- (A) 240 (B) 248 (C) 256 (D) 264 (E) 272
12. Rectangles R_1 and R_2 , and squares S_1 , S_2 , and S_3 , shown below, combine to form a rectangle that is 3322 units wide and 2020 units high. What is the side length of S_2 in units? (AMC 8 2020, P25)



- (A) 651 (B) 655 (C) 656 (D) 662 (E) 666