Handout 3

1. Number of Divisors of a Number N

The number of divisors for a given number N includes 1 and the number itself.

- First write N as a product of prime numbers: $N = p_1^{e_1} \cdot p_2^{e_2} ... p_n^{e_n}$
- Take each exponent of the primes and add 1 to it: $e_1 + 1, e_2 + 1, ..., e_n + 1$
- Multiply these together and you will get the number of divisors: $\#divisors = (e_1 + 1) \cdot (e_2 + 1)...(e_n + 1)$

Problem:

1. How many divisors has 7, 8, 10, 24, 60?

2. Sum of all divisors of a Number N

To find the sum of all divisors of a number:

- write the prime factorization of the number: $N = p_1^{e_1} \cdot p_2^{e_2} ... p_n^{e_n}$
- for each prime, add up all prime to increasing powers starting from zero and up to the last exponent in the prime factorization: $(1 + p_1 + p_1^2 + ... + p_1^{e_1}), (1 + p_2 + p_2^2 + ... + p_2^{e_2}), ..., (1 + p_n + p_n^2 + ... + p_n^{e_n})$
- multiply these numbers: Sum of divisors $= (1+p_1+p_1^2+...+p_1^{e_1}) \cdot (1+p_2+p_2^2+...+p_2^{e_2})...(1+p_n+p_n^2+...+p_n^{e_n})$

Problems:

1. What is the sum of all divisors of 5? 10? 12? 32?

3. Product of all divisors of a number N

Let d be the number of divisors of N. Then the product of all divisors is: $N^{\frac{d}{2}}$

4. Math Battle with problems from AMC 8 and Mathcounts

1. What is the greatest three digit positive number that is divisible by 5, 7 and 9? (Mathcounts)

2. How many odd divisors does 240 have? How many even divisors does 240 have?

3. What is the smallest positive number with 3 divisors?

4. What is the smallest positive number with 4 divisors?

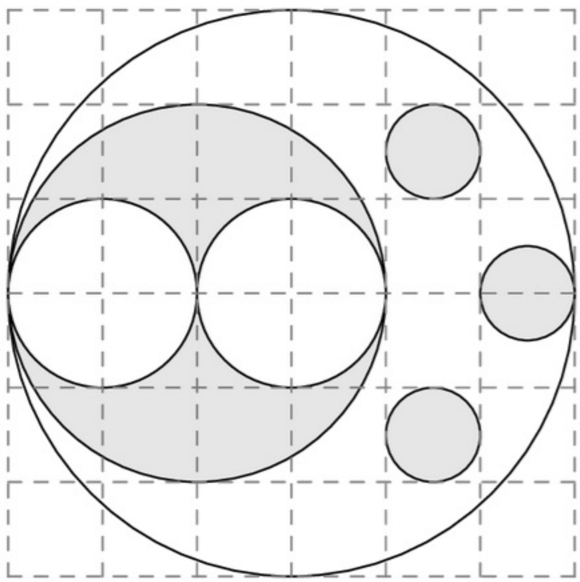
5. If a number has exactly 5 divisors, how many divisors does n^2 have?

6. What is the smallest positive odd integer that has exactly 12 divisors?

7. What is the prime factorization of the product of the divisors of 45?

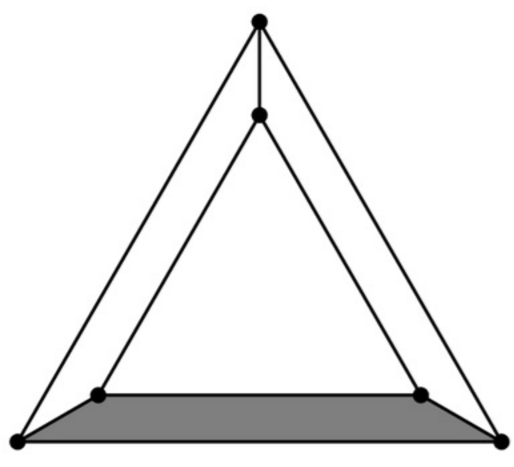
8. A number a has $2b$ factors. What is the product of the factors of a in terms of a and b?

9. The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded? (AMC 8)



(A) $\frac{1}{4}$ **(B)** $\frac{11}{36}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{19}{36}$ **(E)** $\frac{5}{9}$

10. An equilateral triangle is placed inside a larger equilateral triangle so that the region between them can be divided into three congruent trapezoids, as shown below. The side length of the inner triangle is $\frac{2}{3}$ the side length of the larger triangle. What is the ratio of the area of one trapezoid to the area of the inner triangle? (AMC 8)



(A) $1:3$ **(B)** $3:8$ **(C)** $5:12$ **(D)** $7:16$ **(E)** $4:9$