Homework for May 10, 2026.

Algebra/Geometry. Complex numbers.

Please, complete the previous homework assignments. Review the classwork handout on complex numbers. Solve the following problems.

Problems.

1. Compute:
    a. $(2-i)^{-1}$
    b. $\frac{-i}{4\sqrt{3}-i}$
    c. $\frac{1}{3-4i}$
    d. $(1+i)^{-10}$
2. Solve the following equations in complex numbers:
    a. $z^2 = -i$
    b. $z^2 = 2\sqrt{3} + 2i$
    c. $z^2 - z - 1 = 0$
    d. $z^2 + z - 1 = 0$
3. (i) find the magnitude and the argument, (ii) compute the inverse, and (iii) find the magnitude and the argument of the inverse for the following complex numbers:
    a. $1+i$
    b. $-i$
    c. $1+ix$
    d. $\frac{\sqrt{3}}{2}+\frac{i}{2}$
    e. $\frac{1}{2-i}-\frac{1}{2+i}$
4. Find a complex number z whose magnitude is 2 and the argument $Arg(z) = \frac{\pi}{4} = 45°$.
5. Draw the following sets of points on complex plane.
    f. $\{z|Re(z) = 1\}$
    g. $\left\{z|Arg(z) = \frac{3\pi}{4} = 135°\right\}$
    h. $\{z|\ |z| = 1\}$
    i. $\{z|Re(z^2) = 0\}$
    j. $\{z|\ |z^2| = 2\}$
    k. $\{z|\ |z-1| = 1\}$

l. $\{z | z + \bar{z} = 1\}$

6. Prove that for any complex number z, we have
   m. $|\bar{z}|=|z|$, $Arg(\bar{z}) = -Arg(z)$
   n. $\frac{\bar{z}}{z}$ has magnitude 1; check this for $z = 1 - i$.
7. If z has magnitude 2 and argument $\frac{\pi}{2}$ and w has magnitude 3 and argument $\frac{\pi}{3}$, what will be the magnitude and the argument of zw? Write it in the form $a + bi$.
8. Let $P(x)$ be a polynomial with real coefficients.
   o. Prove that for any complex number z, we have $\overline{P(z)} = P(\bar{z})$
   p. Let z be a complex root of this polynomial, $P(z) = 0$. Prove that then $\bar{z}$ is also a root, $P(\bar{z}) = 0$.
9. Solve the equation $x^3 - 4x^2 + 6x - 4 = 0$. Find the sum and product of the roots in two ways: by using Vieta formulas and by explicit computation. Check that the results match.
10. Simplify following expression:

    q. $(1 + \sin\alpha)(1 - \sin\alpha)$
    r. $(1 + \cos\alpha)(1 - \cos\alpha)$
    s. $\sin^4\alpha - \cos^4\alpha$

11. Prove the following equalities:

    t. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
    u. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$
    v. $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$
    w. $\sin 4\alpha = 4\sin\alpha\cos^3\alpha - 4\cos\alpha\sin^3\alpha$
    x. $\sin 5\alpha = 16\sin^5\alpha - 20\sin^3\alpha + 5\sin\alpha$
    y. $\cos 5\alpha = \cdots$ (find the expression)

12. Solve the following equation:

    z. $\cos^2\pi x + 4\sin\pi x + 4 = 0$

13. Solve the following equations and inequalities:
    a. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$
    b. $\cos 3x - \sin x = \sqrt{3}(\cos x - \sin 3x)$
    c. $\sin^2 x - 2\sin x\cos x = 3\cos^2 x$
    d. $\sin 6x + 2 = 2\cos 4x$

e. $\cot x - \tan x = \sin x + \cos x$
f. $\sin x \geq \pi/2$
g. $\sin x \leq \cos x$