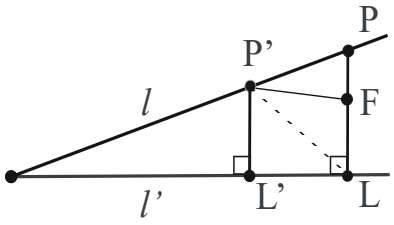
Homework for March 8, 2026 Math 9

Geometry.

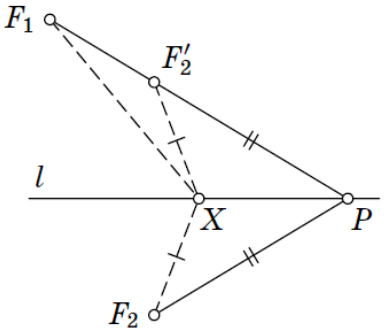
Review the previous classwork notes. Solve the problems below and the remaining problems from the previous homework.

Problems.

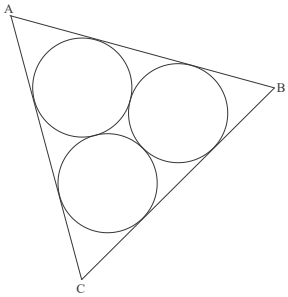
1. Given two lines, l and l', and a point F not on any of those lines, find point P on l such that the (signed) difference of distances from it to l' and F, $|P'L'| - |P'F|$, is maximal. As seen in the figure, for any P' on l the distance to l', $|P'L'| \leq |P'L| \leq |P'F| + |FL|$, where $|FL|$ is the distance from F to l'. Hence, $|P'L'| - |P'F| \leq |FL|$, and the difference is largest ($= |FL|$) when point P belongs to the perpendicular FL from point F to l'.



2. Given line l and points F_1 and F_2 lying on different sides of it, find point P on the line l such that the absolute value of the difference in distances from P to points F_1 and F_2 is maximal. As above, let F_2' be the reflection of F_2 in l. Then for any point X on l, $|XF_2| - |XF_1'| \leq |F_1F_2'|$.



3. Find the (x, y) coordinates of the common (intersection) point of the two lines, one passing through the origin at 45 degrees to the X-axis, and the other passing through the point (1,0) at 60 degrees to it.
4. Find the (x, y) coordinates of the common (intersection) points of the parabola $y = x^2$ and of the ellipse centered at the origin and with major axis along the Y-axis whose length equals 2, and the minor axis along the X-axis whose length equals 1.
5. (Skanavi 10.122) Find the locus of the midpoints of all chords of a given circle with the center O, which intersect given chord AB of this circle.



6. Three circles of radius r touch each other. Find the area of the triangle ABC formed by tangents to pairs of circles (see figure).