May 17, 2026

The final Math battle 9.

1. How many natural numbers <1000 are not divisible by 7, 9 and 13?
2. Pete puts 100 stones in a row: black one, white one, black one, white one, ..., black one, white one. In a single move either Pete chooses two black stones with only white stones between them, and repaints all these white stones in black, or Pete chooses two white stones with only black stones between them and repaints all these black stones in white. Can Pete with a sequence of moves described above obtain a row of 50 black stones followed by 50 white stones?
3. A turtle terrarium is an 8×8 grid of square cells. The turtle can move from any cell to another cell sharing a side with it. A caring owner wants to place water sources in some cells so that from any cell the turtle can reach water in at most three moves. What is the minimum number of water sources needed?
4. What is the maximal number of different integer solutions for x that the following polynomial equation of the seventh order might have?
   $x^7 + a\,x^6 + b\,x^5 + c\,x^4 + d\,x^3 + f\,x^2 + h\,x = 2026$
   Here a, b, c, d, f, h are unknown coefficients.
5. Let $F(x)$ be a function obtained by a sequential application of function $f(x)$ 2026 times, $F(x) = \overbrace{f(f(f(\dots f(f(x))\dots)))}^{2026\ times}$, $x \in \mathbb{R}$. Find all real solutions of the equation:
   a. $F(x) = 2$, where the function $f(x) = \sqrt{\frac{1}{2}x^2 + 2}$
   b. $F(x) = 0$, where the function $f(x) = x^2 + 10x + 20$
6. Let ABC be a right triangle, with $\angle A = 90°$, and $K \in BC$ be such that $AB = AK$. If we know that segment AK bisects the angle bisector CL, what are the angles of ΔABC?

Extra problems.

1. You have 50 beads, all different. How many different necklaces of length 25 can you make using these beads? The clasp on the necklace can be ignored, in the sense that 1-2-clasp-3-4 is the same as 1-clasp-2-3-4.
2. Four friends, A., B., C. and D., decided to exchange presents. They agreed that each one prepares a present, which will then be randomly drawn. Hence, each can get, with equal probability, any of the four presents. What is the probability that no one gets his/her own present, while A. gets the present from D.?
3. A city has 10 bus routes. Is it possible to arrange the routes and the bus stops so that if one route is closed, it is still possible to get from any one stop to any other (possibly changing the route along the way), but if any two routes are closed, there are at least two stops such that it is impossible to get from one to the other?
4. In the number 454**, find the missing digits so that the number is divisible by 2, by 7, and by 9.
5. Is the product, $(2026 \cdot 2027 \cdot 2028 \cdot \ldots \cdot 4052)$, divisible by 2026! (2024 factorial)? What about $(2025 \cdot 2025 \cdot 2026 \cdot \ldots \cdot 4050)$?
6. Prove that the product of any m consecutive integer numbers is divisible by $m!$
7. Given an equilateral triangle ABC, find all points M on the plane such that both triangles ABM and ACM are isosceles.