MATH 8 [09/14/2025]
HANDOUT 1: ALGEBRA AND GEOMETRY REVIEW TEST

Algebra review

1. Open parentheses and expand the following expressions:
   (a) $(a+b)^2 =$
   (b) $(a-2b)^2 =$
   (c) $(3a+b)^3 =$
   (d) $(a-b)^3 =$

2. Factor the following expressions:
   (a) $a^2 - b^2 =$
   (b) $a^3 + b^3 =$
   (c) $a^2 + b^2 =$
   (d) $a^4 + 4b^4 =$
   (e) $1 + a + a^2 + a^3 =$

3. Solve the following equations
   (a) $x + 1/x = 5.2$
   (b)

$$\frac{x^2+1}{x} - \frac{2x}{x^2+1} = 1$$

4. If α and β are the roots of the equation $x^2 = x + 1$, find $\frac{1}{\alpha} + \frac{1}{\beta}$.

5. Solve the following inequality (write your answer as a set of possible values for x):

$$\frac{(x+1)(x-7)(x+2)^2}{(x^2+1)(x+3)} \leq 0.$$

6. What is the coefficient of x^3y^7 in $(2x-y)^{10}$? What about the coefficient of x^5y^4?

7. We have k books and n backpacks. How many ways are there of distributing them in these scenarios?
   (a) Books and backpacks are all distinct, we can place any number of books in any backpack (including 0).
   (b) Books and backpack all distinct, $k \leq n$, and we can put at most one book in any backpack.
   (c) Books are identical, backpacks are distinct, and we can put an arbitrary number of books in any backpack (including 0).
   (d) Books are identical, backpacks are distinct, $k \geq n$, and we can put an arbitrary number of books any backpack (but at least 1).
   (e) Books are identical, backpacks are distinct, $k \leq n$, and we can put at most one book in any backpack.
   (f) Make up your own constraints similar to any of the above, and see if you can solve it!

8. Eight players A through H have reached the quarter finals of the women's US open tennis championship.

(a) How many ways are there to pair them up to play the quarter final games against each other?
(b) How many ways are there to draw up the tournament roster from the quarter finals on (i.e. including the semi-finals and finals)?
(c) How many different outcomes are possible for the first and second prizes?

9. What is the remainder when 3^{2025} is divided by 23?

10. Describe all integers which have a remainder of 2 when divided by 7 and a remainder of 5 when divided by 13.

Geometry Review

1. Four equal segments are cut off a circle of radius r so that a square is obtained. Find the area of each of these segments.

2. O is the center of the inscribed circle in triangle ABC. The angle AOB is $135°$. Find the angle ACB.

3. Trapezoid $ABCD$ is inscribed in a circle of radius r, such that AB is a diameter, and the length of CD is r. Find the area of the trapezoid.

4. Given a triangle ABC, with $AB = 5cm$, $AC = 4cm$, and $\angle CAB = 45°$, show how to find a point D on $\overrightarrow{AC}$ extended, such that $AD = 2BD$.

5. Is there a triangle with sides
(a) $5cm$, $3cm$, $6cm$?
(b) $5cm$, $3cm$, $9cm$?

6. When can a parallelogram
(a) have an inscribed circle?
(b) be inscribed in a circle?

7. Given a circle λ with center O and radius r, and a point A outside it, show how to draw a tangent from A to λ.