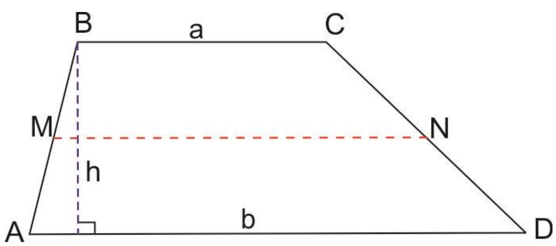
Homework due on September 28, 2025.

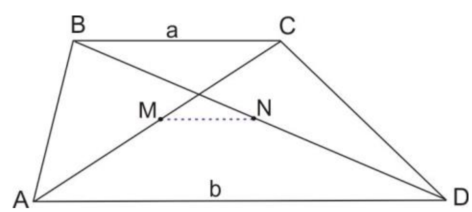
# Geometry.

Review the classwork handout. In all the problems, you are only allowed to use theorems you had proved before. The words "construct..." mean "describe how this can be constructed using ruler and compass". In these problems, you can freely reuse the constructions which were done before - e.g., you can just say "construct the perpendicular from this point to this line", without repeating how it can be done - you have already discussed it once, so there is no need to repeat it.

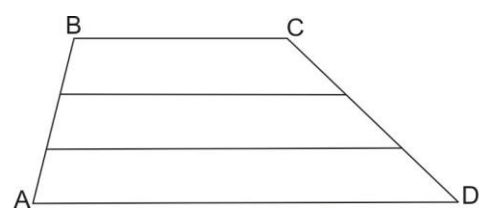
**Problems.**

1. Midsegment of a triangle is a line segment joining the midpoints of two sides. Prove that midsegment of a triangle is parallel to the third side and its length is 1/2 of that side's length.
2. A *trapezoid* is a quadrilateral with two parallel sides. An *isosceles trapezoid* is a trapezoid in which base angles are equal and therefore the lengths of the left and the right side are also equal. Prove that the area of a trapezoid is $S = h\frac{a+b}{2}$.




3. Prove that segment connecting midpoints of the diagonals of a trapezoid is parallel to its bases and that the length of this segment is half of the difference of bases, $|MN| = \frac{b-a}{2}$.




4. Two lines parallel to trapezoid's bases divide sides into segments of equal length. Bases of the trapezoid are 2 and 5 sm. What are the lengths of other parallel segments?
5. Using a ruler and a compass, construct a triangle given the midpoints of its three sides.

# Algebra.

Review the classwork handout. Solve the following problems.

1. Let $P$, $Q$ and $R$ be some logical propositions. Write a negation of
   a. $((P \wedge Q) \vee R)$
   b. $(P \wedge (Q \vee R))$
   c. $(P \Rightarrow (Q \vee R))$
   d. $(P \Rightarrow (Q \Leftrightarrow R))$
   e. $(P \Leftrightarrow (Q \Rightarrow R))$
2. Solve the following equation, writing your solution with properly tracking the equivalence of the transformed forms,

$$\frac{x^2-3x+2}{x} + \frac{x}{x^2-3x+2} = 2.5$$

3. Prove that

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} = \frac{a-c}{b-d} = \frac{a+c}{b+d}.$$

Consider how this property is used in proving the Thales theorem.

4. Factor the following expressions
   a. $1 + a + a^2 + a^3 + a^4 + a^5$
   b. $1 + a - 2a^2 - 2a^3 + a^4 + a^5$
5. Simplify expressions:
   a. $\frac{x+y}{x} - \frac{x}{x-y} + \frac{y^2}{x^2-xy}$
   b. $\frac{\sqrt{1+\left(\frac{x^2-1}{2x}\right)^2}}{(x^2+1)\frac{1}{x}}$
   c. $\frac{\sqrt{2b+2\sqrt{b^2-4}}}{\sqrt{b^2-4}+b+2}$
6. Find all natural numbers $a$ and $b$, such that $a^3 - b^3 = 19$
7. Find the roots of the following equation, writing your solution with properly tracking the equivalence of the transformed forms,

$$\frac{14}{20-6x-2x^2} + \frac{x^2+4x}{x^2+5x} - \frac{x+3}{x-2} + 3 = 0.$$