Homework (Geometry)
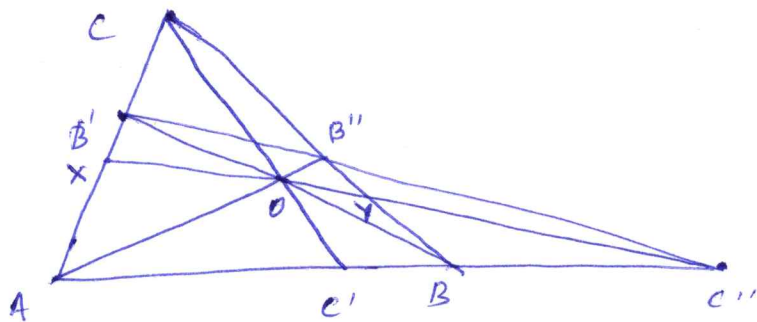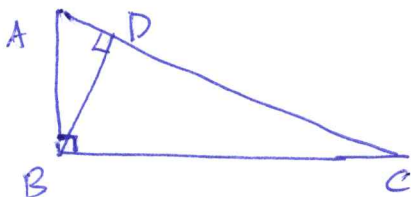
1. a) Prove $(A,B; C,D) = (C,D; A,B)$

   b) Prove $(A,C; B,D) = 1 - (A,B; C,D)$

   c) Prove that if A,B,C,D are distinct points on a line, and $(A,B; C,D) = (B,A; C,D)$ then $(A,B; C,D) = -1$

2. In the picture below, complete the proof that $(A,B; C',C'') = -1$ as follows, by justifying each line

$$(A,B; C',C'') = (X,Y; O,C'') \qquad \leftarrow \text{draw 4 lines through } C$$
$$= (C,Y; B'',B) \qquad \leftarrow \text{lines through } A$$
$$= (X,Y; C'',O) \qquad \leftarrow \text{lines through } B'$$
$$= (A,B; C'',C') \qquad \leftarrow \text{lines through } C$$

3. Prove that in a right triangle, each side of the right angle is the geometric mean between the hypotenuse and its projection onto the hypotenuse. That is, if BD is the altitude from the vertex of the right angle $\angle ABC$, on to the hypotenuse AC, then $|AB|^2 = (AC) \cdot |AD|$

4. Prove that the three medians in a triangle divide it into six smaller triangles of equal area.

Algebra Homework 8

1. Using Euclid's algorithm, provide the continued fraction representation for each of the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1st, 2nd, 3rd, ... level with the value of the number itself (in decimal representation). $\dfrac{1351}{780}$, $\dfrac{25344}{8069}$, $\dfrac{29376}{9347}$, $\dfrac{6732}{1785}$, $\dfrac{2187}{2048}$, $\dfrac{3125}{2401}$

2. Is there a number, x, represented by the following infinite continued fraction? If so, find it.

   a. $$x = 5 - \cfrac{6}{5 - \cfrac{6}{5 - \cfrac{6}{5 - \dots}}}$$

   b. $$x = 2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \dots}}}$$

   c. $$x = 1 - \cfrac{6}{1 - \cfrac{6}{1 - \cfrac{6}{1 - \dots}}}$$

3. Write the first few terms in the following sequence ($n \geq 1$),

   $$n\,fractions \begin{cases} \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}} = ? \\ \qquad \dots + \cfrac{1}{1 + x} \end{cases}$$

   a. Try guessing the general formula of this fraction for any n.

   b. Using mathematical induction, try proving the formula you guessed.

4. Can you prove that

   a. $\dfrac{3 + \sqrt{17}}{2} = 3 + \cfrac{2}{3 + \cfrac{2}{3 + \cfrac{2}{3 + \dots}}}$?

   b. $1 = 3 - \cfrac{2}{3 - \cfrac{2}{3 - \cfrac{2}{3 - \dots}}}$?

   c. $\dfrac{4}{2 + \cfrac{4}{2 + \cfrac{4}{2 + \dots}}} = 1 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \dots}}}$? Find these numbers.