Homework for January 18, 2026 (due Jan 25)

Algebra.

Review the classwork handout. Solve the remaining problems from the previous homework assignments and classwork exercises. Try solving the following problems.

1. Using the inclusion-exclusion principle, find how many natural numbers $n < 100$ are not divisible by 3, 5 or 7.
2. Four letters a, b, c, d, are written down in random order. Using the inclusion-exclusion principle, find probability that at least one letter will occupy its alphabetically ordered place? What is the probability for five letters?
3. Using the inclusion-exclusion principle, find the probability that if we randomly write a row of digits from 0 to 9, no digit will appear in its proper ordered position.
4. Secretary prepared 5 different letters to be sent to 5 different addresses. For each letter, she prepared an envelope with its correct address. If the 5 letters are to be put into the 5 envelopes at random, what is the probability that
    a. no letter will be put into the envelope with its correct address?
    b. only 1 letter will be put into the envelope with its correct address?
    c. only 2 letters will be put into the envelope with its correct address?
    d. only 3 letters will be put into the envelope with its correct address?
    e. only 4 letters will be put into the envelope with its correct address?
    f. all 5 letters will be put into the envelope with its correct address?
5. Among 24 students in a class, 14 study mathematics, 10 study science, and 8 study French. Also, 6 study mathematics and science, 5 study mathematics and French, and 4 study science and French. We know that 3 students study all three subjects. How many of these students study none of the three subjects?
6. In a survey on the students' chewing gum preferences, it was found that
    a. 20 like juicy fruit.
    b. 25 like spearmint.
    c. 33 like watermelon.
    d. 12 like spearmint and juicy fruit.
    e. 16 like juicy fruit and watermelon.
    f. 20 like spearmint and watermelon.
    g. 5 like all three flavors.
    h. 4 like none.

    How many students were surveyed?

Geometry.

1. Review the solution of the radical axis of two circles problem: find the locus of points whose powers with respect to two non-concentric circles are equal. Consider situation when circles are concentric.

2. Complete the following exercises from class. Find the locus of points satisfying each of the following equations or inequalities (graph it on a coordinate plane).

   a. $|x| = |y|$

   b. $|x| + x = |y| + y$

   c. $|x|/x = |y|/y$

   d. $[y] = [x]$

   e. $\{y\} = \{x\}$

   f. $x^2 - y^2 \geq 0$

   g. $x^2 + y^2 \leq 1$

   h. $x^2 + 8x = 9 - y^2$

3. Describe the locus of all points (x, y) equidistant to the X-axis (i. e. the line $y = 0$) and a given point $P\ (0, 2)$ on the Y-axis. Write the formula relating y and x for these points.
4. Find the (x, y) coordinates of the vertex C of an equilateral triangle ABC if A and B have coordinates $A(1, 3)$ and $B(3, 1)$, respectively.
5. Find the (x, y) coordinates of the vertices C and D of a square $ABCD$ if A and B have coordinates $A(2, 1)$ and $B(4, 0)$, respectively.