MATH 9B: HOMEWORK 14 (ASSIGNED FEB 1; DUE FEB 8)

1. Algebra

1. Present examples of binary relations that are, and that are not equivalence relations.

2. For each of the following relations, check whether it is an equivalence relation and describe all equivalence classes.

(a) On $\mathbb{R}$: relation given by $x \sim y$ if $|x| = |y|$.

(b) On $\mathbb{Z}$: relation given by $a \sim b$ if $a \equiv b \mod 5$.

(c) On $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$; describe the equivalence class of $(1, 2)$.

(d) Let $\sim$ be the relation on the set of all directed segments in the plane defined by $\overrightarrow{AB} \sim \overrightarrow{A'B'}$ if $ABB'A'$ is a parallelogram.

(e) On the set of pairs of integers, $\{(a, b), a, b \in \mathbb{Z}, b \neq 0\}$, the relation given by $(a_1, b_1) \sim (a_2, b_2)$ if $a_1 b_2 = a_2 b_1$. Describe these equivalence classes.

3. Let $f : X \longrightarrow Y$ be a function. Define a relation on X by $x_1 \sim x_2$ if $f(x_1) = f(x_2)$. Prove that it is an equivalence relation. Describe the equivalence classes for the equivalences defined by the following functions on R.

(a) $f(x) = x^2$, i.e., $x \sim y$ if $x^2 = y^2$.

(b) $f(x) = \sin x$. i.e. $x \sim y$ if $\sin x = \sin y$.

2. Geometry

Review the notes from this and the previous week's classwork.

1. Review derivation of the equation describing an ellipse and derive in a similar way,

(a) The equation of an ellipse, defined as the locus of points P for which the distance to a given point (focus F_2) is a constant fraction of the perpendicular distance to a given line called the directrix, i.e., $|PF_2|/|PD| = e < 1$.

(b) Equation of a hyperbola, defined as the locus of points for which the ratio of the distances to one focus and to a line (called the directrix) is a constant e. However, for a hyperbola it is larger than 1, $|PF_2|/|PD| = e > 1$.

2. Find (describe) set of all points formed by the centers of the circles that are tangent to a given circle of radius r and a line at a distance $d > r$ from its center O.

3. Using the method of coordinates, prove that the geometric locus of points from which the distances to two given points have a given ratio, $q \neq 1$, is a circle.

4. Find the equation of the locus of points equidistant from two lines, $y = ax + b$ and $y = mx + n$, where a, b, m, n are real numbers.

5. Find the distance between the nearest points of the circles,

(a) $(x - 2)^2 + y^2 = 4$ and $x^2 + (y - 1)^2 = 9$.

(b) $(x + 3)^2 + y^2 = 4$ and $x^2 + (y - 4)^2 = 9$

(c) $(x - 2)^2 + (y + 1)^2 = 4$ and $(x + 1)^2 + (y - 3)^2 = 5$

(d) $(x - a)^2 + y^2 = r_1^2$ and $x^2 + (y - b)^2 = r_2^2$.