Homework for December 14, 2025.

Algebra.

Review the classwork handout. Solve the remaining problems from the previous homework assignments and classwork exercises. Try solving the following problems (some problems are repeated from previous homework – skip the ones you have already solved).

1. Rewrite the following properties of set algebra and partial ordering operations on sets in the form of logical propositions, following the first example.
   a. $[\mathrm{A} \cdot (\mathrm{B} + \mathrm{C}) = \mathrm{A} \cdot \mathrm{B} + \mathrm{A} \cdot \mathrm{C}] \Leftrightarrow \left[(\mathrm{x} \in \mathrm{A}) \wedge \big((\mathrm{x} \in \mathrm{B}) \vee (\mathrm{x} \in \mathrm{C})\big)\right] = \left[\big((\mathrm{x} \in \mathrm{A}) \wedge (\mathrm{x} \in \mathrm{B})\big) \vee \big((\mathrm{x} \in \mathrm{A}) \wedge (\mathrm{x} \in \mathrm{C})\big)\right]$
   b. $A + (B \cdot C) = (A + B) \cdot (A + C)$
   c. $(A \subset B) \Leftrightarrow A + B = B$
   d. $(A \subset B) \Leftrightarrow A \cdot B = A$
   e. $(A + B)' = A' \cdot B'$
   f. $(A \cdot B)' = A' + B'$
   g. $(A \subset B) \Leftrightarrow (B' \subset A')$
   h. $(A + B)' = A' \cdot B'$
   i. $(A' + B')' + (A' + B\,)' = A$
2. Using definitions from the classwork handout, devise logical arguments proving each of the following properties of algebra and partial ordering operations on sets and draw Venn diagrams where possible (hint: use problem #1).
   a. $\mathrm{A} \cdot (\mathrm{B} + \mathrm{C}) = \mathrm{A} \cdot \mathrm{B} + \mathrm{A} \cdot \mathrm{C}$
   b. $A + (B \cdot C) = (A + B) \cdot (A + C)$
   c. $(A \subset B) \Leftrightarrow A + B = B$
   d. $(A \subset B) \Leftrightarrow A \cdot B = A$
   e. $(A + B)' = A' \cdot B'$
   f. $(A \cdot B)' = A' + B'$
   g. $(A \subset B) \Leftrightarrow (B' \subset A')$
   h. $(A + B)' = A' \cdot B'$
   i. $(A' + B')' + (A' + B\,)' = A$
3. Verify that a set of eight numbers, {1,2,3,5,6,10,15,30}, where addition is identified with obtaining the least common multiple,

$$m + n \equiv LCM(n, m)$$

multiplication with the greatest common divisor,

$$m \cdot n \equiv GCD(n, m)$$

$m \subset n$ to mean "m is a factor of n",

$$m \subset n \equiv \left(n = 0 mod(m)\right)$$

and

$$n' \equiv 30/n$$

satisfies all laws of the set algebra.

4. For a set A, define the characteristic function χ_A as follows,

$$\chi_A(x) = \begin{cases} 1, if\ x \in A \\ 0, if\ x \notin A \end{cases}$$

Show that χ_A has following properties

$$\chi_A = 1 - \chi_{A'}$$

$$\chi_{A \cap B} = \chi_A \chi_B$$

$$\chi_{A \cup B} = 1 - \chi_{A' \cap B'} = 1 - \chi_{A'} \chi_{B'} = 1 - (1 - \chi_A)(1 - \chi_B)$$
$$= \chi_A + \chi_B - \chi_A \chi_B$$

Write formulas for $\chi_{A \cup B \cup C}, \chi_{A \cup B \cup C \cup D}$.

5. Consider the quadratic equation $x^2 = 7x + 1$. Find a continued fraction corresponding to a root of this equation.

6. Using the continued fraction representation, find rational number, r, approximating $\sqrt{2}$ to the absolute accuracy of 0.0001.

7. Consider the values of the following expression, y, for different x. How does it depend on x when n becomes larger and larger?

$$n\ fractions \left\{ \quad y = 3 - \cfrac{2}{3 - \cfrac{2}{3 - \cfrac{2}{3 - \cdots}}} \quad \cdots - \frac{2}{3 - x} \right. .$$