MATH 8: EUCLIDEAN GEOMETRY 7

MARCH 1, 2026

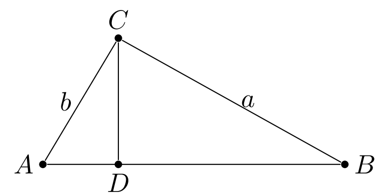
Homework

1. (Pythagorean theorem)Let ABC be a right triangle, $\angle C = 90°$, and let CD be the altitude. Denote by a, b, c the lengths of the sides of this triangle: $a = BC$, $b = AC$, $c = AB$.

(a) Show that triangles $\triangle ACD$, $\triangle ABC$ are similar and deduce from this that $AD = b^2/c$.

(b) Similarly, prove that $BD = a^2/c$

(c) Prove $c^2 = a^2 + b^2$.



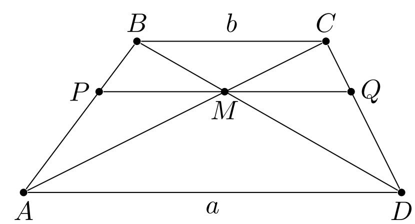
[Pythagorean theorem has dozens of proofs, but most of them are based on the notion of area and thus can't be considered completely rigorous until we give a definition of area and prove properties we normally take for granted, such as that when we cut a figure in two, the area of the original figure is equal to sum of areas of the pieces. These are actually rather hard to prove. The proof above, based on similar triangles, is one of the shortest *rigorous* proofs.]

2. Consider the trapezoid with bases $AD = a$, $BC = b$. Let M be the intersection point of diagonals, and let PQ be the segment parallel to the bases through M.

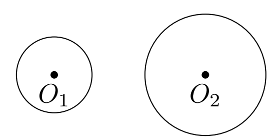
(a) Show that point M divides each of diagonals in proportion $a : b$, e.g. $AM : MC = a : b$.

(b) Show that points P, Q divide sides of the trapezoid in proportion $a : b$.

(c) Show that $PQ = \frac{2ab}{a+b}$. [Hint: compute PM, MQ separately and add.]



3. Given two circles with centers O_1, O_2 and radiuses r_1, r_2 respectively, construct (using straightedge and compass) a common tangent line to these circles. You can assume that circles do not intersect: $O_1O_2 > r_1 + r_2$ and that $r_2 > r_1$.



[Hint: assume that we have such a tangent line, call it l. Then distance from that line to O_1 is r_1, and distance to O_2 is r_2. Thus, if we draw a line l' parallel to l but going through O_1, the distance from l' to O_2 is ... and thus l' is tangent to ...]