May 17, 2026

Math 8 final test.

1. If we want to choose a president, vice-president, and three Committee members from a 100-student class, in how many ways can we do this?
2. What is the sum of all numbers in the n-th row of Pascal's triangle? Are there any rows in the Pascal triangle where all numbers are odd? Which rows are these?
3. Show that if a, b are integers, then $(a + b)^p - a^p - b^p$ is divisible by p.
4. Using binomial formula, prove that for any integer $n \geq 0$, $\forall\, x \geq 0, (1 + x)^n \geq 1 + nx$
5. In a group of 25 people, what are the chances that no two of them have their birthday on the same day? Conversely, what is the chance that at least two people have the same birthday?
6. The logic operation NAND is defined by A NAND $B \Leftrightarrow$ NOT (A AND B)
   a. Write a truth table for NAND
   b. What is A NAND A?
   c. Show that you can write NOT A, A AND B, A OR B using only NAND (possibly using each of A, B more than once)
7. How many different prime factors does 2025^{2025} have? How many prime factors does it have in total (that is, when decomposed into a product of primes, how many prime factors, some of them being equal, are there in that product)?
8. Prove that in any triangle, the three angle bisectors intersect at a single point (compare with the similar fact about perpendicular bisectors).
9. Can you inscribe a circle into a quadrilateral with sides (in order)
   a. 2cm, 2cm, 3cm, 3cm?
   b. 5cm, 3cm, 1cm, 3cm?
   c. 2cm, 5cm, 3cm, 4cm?
10. Prove that equation $p^2 = 3q^2$ has no integer solutions.

Bonus problems.

11. 20 chess players meet for a championship. On the first day, each player played one game. On the second day, each player also played one game (possibly with the same player (s)he played the first day). Is it true that after these two days it is always possible to choose 10 players so that no two of them played against each other?
12. Let us call a number between 0 and 999999 "lucky" if the sum of the first 3 digits is equal to the sum of the last 3 digits (we write all numbers as 6-digit numbers, adding zeros in front, if necessary, e.g., writing 17 as 000017). Is the total number of lucky numbers even or odd?
13. Alice and Bob are playing the following game. They have a staircase with 1001 steps; on some of these steps, there are stones (no more than one stone on each step). At her turn, Alice can take any stone and move it to the nearest free step above it. After that, Bob can take any stone such that the step immediately below is empty and roll this stone one step down. Initially there are 500 stones occupying steps 1-500. Alice goes first; her goal is to get a stone to step 1001. Bob's goal is to prevent Alice from achieving this. Is there a winning strategy for one of them? If so, what is this strategy?
14. Is it possible to put in each vertex of any given triangle ABC a number such that for every edge, its length is equal to the sum of numbers at its endpoints?
15. Let ABC be a right triangle, with $\angle A = 90°$, and $K \in BC$ be such that $AB = AK$. If we know that segment AK bisects the angle bisector CL, what are the angles of ΔABC?
16. Given a circle and a point P inside this circle, construct a chord AB through P such that $|AP| - |BP| = 2\text{cm}$.