March 8, 2026 Math 8

Handout 21. Euclidean Geometry 10: Construction with ruler and compass.

Construction with ruler and compass.

It is sufficient to know how to construct the following, using only a straightedge and a compass:

- construct a segment equal to another
- construct an angle equal to another
- bisect a segment
- bisect an angle
- construct a line perpendicular to another
- construct a line parallel to another.

Combining these elements, one can construct triangles, quadrilaterals, and other figures based only on description and measures. However, this is only the first part of the solution; proving that the figure constructed is unique is the second part. Sometimes the result is not unique, so we have to construct all possible solutions.

To find all solutions, it is useful to think of sets of all points (commonly, a line, a line segment, or a curve), whose location satisfies or is determined by one or more specified conditions. Such a set is called a “locus” (plural “loci”). Some useful loci are,

- bisector of angle $\angle AOB$: locus of points equidistant from lines (OA), (OB);
- line l parallel to line a: locus of points at the same fixed distance from a, in the same half-plane;
- circle $\omega(O; R)$: locus of points at distance R from a single point O;
- arc $\widehat{A\alpha B}$ of circle $\omega(O; R)$: locus of points C such that $\angle ACB = \frac{1}{2}\angle AOB$.

Recall how to find the center of the circle inscribed in ΔABC: it is the intersection of loci,

- equidistant from sides AB and AC (bisector of $\angle A$)
- equidistant from sides BC and AB (bisector of $\angle B$)
- equidistant from sides AC and BC (bisector of $\angle C$)

Since all these loci have a single common point, solution exists and is unique.

Exercise 1: Using only a straightedge and compass, construct a right triangle given two segments equal to a leg and the hypotenuse.

Exercise 2: Using only a straightedge and compass, construct a triangle given a segment equal to a side and a segment equal to the altitude to that side.

Exercise 3: Using only a straightedge and compass, construct a triangle given two angles and a segment equal to its perimeter.

Quiz problems

Solve as many problems as you can. The remaining problems will be assigned as homework.

1. Prove that if two medians of a triangle are congruent, then the triangle is isosceles.
2. Prove that if a median and an angle bisector of a triangle coincide, then the triangle is isosceles.
3. In triangle ABC with $|AB| < |BC|$, draw the altitude BD, the angle bisector BE, and the median BF (with $D, E, F \in AC$). Prove that E lies between D and F. Equivalently, show that
   a. $|AE| > |AD|$
   b. $|AE| < |AF|$
4. *Prove that if two triangles have two sides and a median congruent, then the triangles are congruent (consider two cases, ie, both possibilities for which side the median is drawn to).
5. *Prove that in a right triangle, the bisector of the right angle also bisects the angle between the median and the altitude drawn to the hypotenuse.
6. How many diagonals can be drawn
   a. in a convex octagon?
   b. In a convex n-gon (a polygon with n sides)?
7. Pairs of points A and A' and B and B' are symmetric with respect to line l. Prove that these points are either concyclic (lie on a circle) or collinear (lie on a line).
8. How many interior angles of a convex octagon can be acute?
9. In a quadrilateral $ABCD$ circumscribed about a circle with center O, what is the sum of $m\angle AOB$ and $m\angle COD$?
10. Using only a straightedge and compass, construct a 30º angle; a 45º angle.
11. Given segments of length a, b, and 1 use only a straightedge and compass to construct
    a. Segment of length ab
    b. Segment of length a/b
12. Given segment AB and a line l intersecting AB, use only a straightedge and compass to construct a triangle ABC such that the bisector of $\angle ACB$ lies on line l.